恒星進化論
(Stellar Evolution Theory)

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金曜3限（13:00–14:45）

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講義用 Zoom URL（使うときは）:
https://u-tokyo-ac-jp.zoom.us/j/86808825013?pwd=eOwA1eTsTTdw3quqcrYvUPRen9F6Tb.1

参考図書

*The Physics of Stars,
A.C. Phillips (Wiley)

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*星の進化、斉尾英行（培風館）

*Principles of Stellar Evolution
And Nucleosynthesis
D. D. Clayton (Chicago)

*Supernovae and Nucleosynthesis
A. Arnett (Princeton)

*Physics, Formation and Evolution
of Rotating Stars, A. Maeder
(Springer)

*Black Holes, White Dwarfs, and
Neutron Stars, S. Shapiro,
S.Teukolsky
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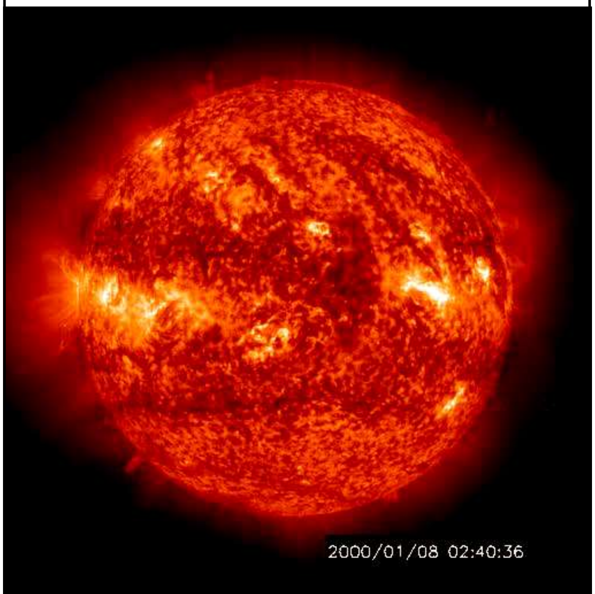
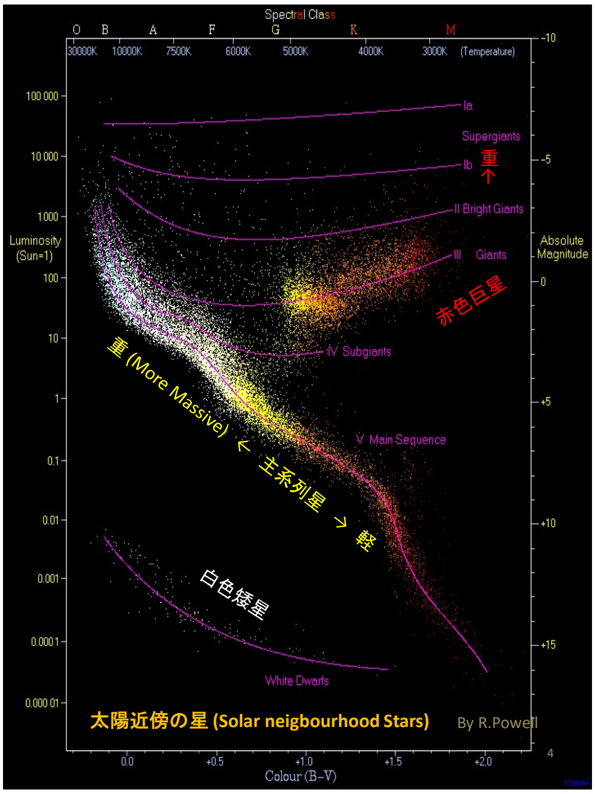
*元素はいかにつくられたか
超新星爆発と宇宙の化学進化
野本憲一　編（岩波書店）

*超新星（新天文学ライブラリー4）
山田章一　（日本評論社）

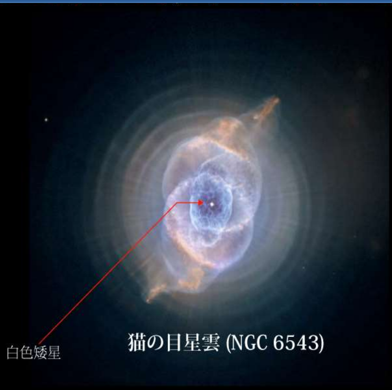
Sun



3



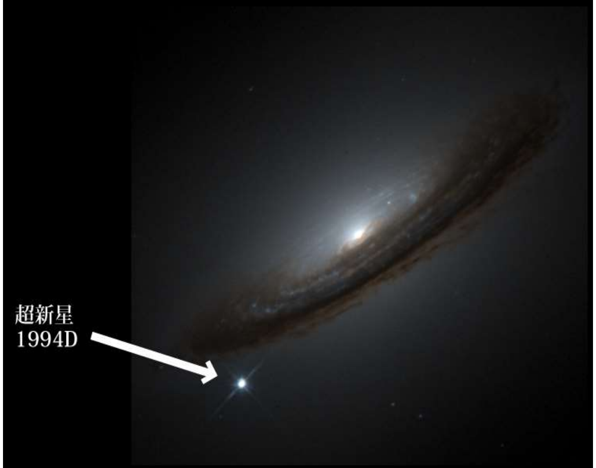
Planetary nebula and a White dwarf



ハッブル宇宙望遠鏡撮影

5

Supernova (Type Ia)

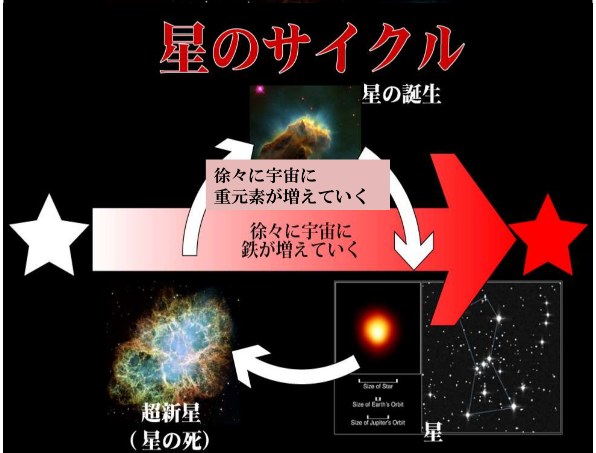


6

Supernova explosion simulation (Core collapse SN)

t = 8.0e−01 s

Supernova explosion
& Nucleosynthesis
Formation of Stars
Evolution of Stars



8

恒星の進化論

- 星は誕生から様々に姿を変え、宇宙に元素をばらまいていく
- この講義では、これら星の進化にまつわる現象を紹介し、それを理解するために基礎となる物理や、実際に計算を行うための手法（数値計算法を含む）を解説する
- また研究の最前線や現在問題となっている事柄も紹介する

前半の重点項目

- 恒星進化の基礎方程式
- 星のポリトロープモデル
- 対流発生の条件
- 恒星はなぜ赤色（超）巨星になるのか
- 混合距離（ミキシング・レングス）理論

星の進化論の基礎
(Basic of Stellar Evolution Theory)

星の進化計算

- 1次元球対称の星の準静的進化
- 基本となる方程式：
  - 連続の式
  - 静水圧平衡の式
  - 状態方程式　P(ρ、T)
  - エネルギー伝播の式
    - 放射、及び対流
  - エネルギー生成と化学進化
    - 核反応ネットワーク
  - エネルギー保存の式
  - Opacity

大質量星
- ニュートリノによるエネルギー損失
- 質量放出（Mass　Loss)

星の進化計算

- 基本となる方程式：１次元球対称
  - 連続の式 $\frac{dm}{dr} = 4\pi r^2 \rho$ (1.1) (KH4, 10)
  - 静水圧平衡の式 $\frac{dP}{dr} = -\frac{Gm\rho}{r^2}$ (1.2)
  - 状態方程式(EOS) P(ρ、T)

    理想気体 $P = \frac{k}{\mu m_u} \rho T$ (1.3)

    完全縮退したガス $P \propto \rho^{5/3} (non-relativistic)$

    $P \propto \rho^{4/3} (relativistic)$
  - 特殊な場合（例、ρ一定、EOSが温度に依存しない場合）は上記の方程式を残りの温度進化の式と独立に解くことができる
  - 特にEOSが $P = K\rho^{1+\frac{1}{n}}$ (1.4) と書ける場合 n をポリトロープ指数と呼び、解は polytropic stellar models と呼ばれる

残りの式
- エネルギー生成と化学進化 （核反応ネットワーク）
- エネルギー伝播の式 （放射、及び対流）
- Opacity
- エネルギー保存の式

静水圧平衡の式とビリアル定理

$$(1.2) \rightarrow \frac{dP}{dr} = -\frac{Gm\rho}{r^2}$$

$$\int_0^R 4\pi r^3 \frac{dP}{dr} dr = -\int_0^R \frac{Gm\rho}{r} 4\pi r^2 dr = -\int_0^M \frac{Gm}{r} dm = E_{GR}$$

左辺（部分積分） $= -3\int_0^R P(r) 4\pi r^2 dr = -3 < P > V$

$$< P > = -\frac{E_{GR}}{3V} \quad (1.5)$$

気体運動学（理想気体、非相対論的ガス）

$$< P > = \frac{n}{3} < \mathbf{p} \bullet \mathbf{v} > = \frac{2n}{3} < \frac{1}{2} mv^2 > = \frac{2}{3} \frac{E_{KE}}{V}$$

$(1.5) \rightarrow 2E_{KE} + E_{GR} = 0$ （1.6 ビリアル定理）

nは単位体積当たりの粒子数

（相対論的ガスの場合は $< P > = \frac{n}{3} < pc > = \frac{1}{3} \frac{E_{KE}}{V}$

なので $E_{KE} + E_{GR} = 0$ となる）

太陽の平均密度、温度

星の平均密度 $< \rho > \approx \frac{M}{\frac{4\pi}{3} R^3}$ ($= 1.4 g / cm^3$ 太陽の場合)

$M_\odot$ =1.99・10^{33} g, $R_\odot$ =6.96・10^{10} cm,
G=6.67 ・10^{-8} $cm^3/(g\ s^2)$

$$E_{GR} = -\int_0^R \frac{Gm\rho}{r} 4\pi r^2 dr = -\frac{3GM^2}{5R} \quad (\rho = 一定の時)$$

一般に $E_{GR} \approx -\frac{GM^2}{R}$ （1.7）

$$(1.5) \rightarrow < P > \approx \frac{GM^2}{4\pi R^4}$$

理想気体 EOS を使うと $< T > = \frac{GM}{3R} \frac{\mu m_u}{k}$

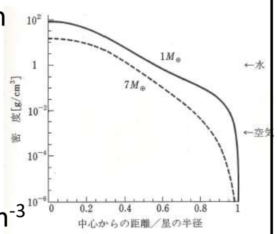
$= 4.7 \cdot 10^6$ K（太陽）

平均分子量 μ = 0.62
(X=0.7, Y=0.28, Z=0.02)

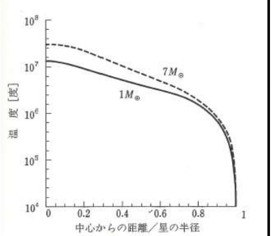
太陽のPolytropicモデル

$P = K\rho^{1+\frac{1}{n}}$ (polytropic relation)

- 理想気体であってもTとPにある関係がある時にはpolytropic となる
  - d ln T/ d ln P =1/(n+1) のとき
- Lane-Emden equation



- n=3 polytrope
  - ρ_c/<ρ> = 54.18
  - ρ_c= 54.18*1.4 g cm^{-3}
    = 76.4 g cm^{-3}
  - T_c= 1.4 x 10^7 K
  - より詳しい計算値
    1.5x 10^7 Kに近い



Polytrope & White dwarfs

$$K = c_1 A^{-2} G \rho_c^{\frac{n-1}{n}} \quad \text{(KH 19.9)},$$

c_i : non dimensional constants

$$\& \ \rho_c \propto \bar{\rho} \propto MR^{-3} = M\left(\frac{A}{z_n}\right)^3$$

$$\rightarrow A = c_2 \rho_c^{1/3} M^{-1/3}$$

$$\rightarrow K = c_3 M^{2/3} G \rho_c^{1-\frac{1}{n}-\frac{2}{3}} = c_3 M^{2/3} G \rho_c^{\frac{1}{3}-\frac{1}{n}},$$

$$\text{also}: P_c = K\rho_c^{1+\frac{1}{n}} = c_3 G M^{2/3} G \rho_c^{\frac{4}{3}}$$

$$P_c = 0.48 \ GM^{2/3} G \rho_c^{\frac{4}{3}} \qquad (n = \frac{3}{2}) \quad (1.21)$$

$$= 0.36 \ GM^{2/3} G \rho_c^{\frac{4}{3}} \qquad (n = 3) \quad (1.22)$$

Lane-Emden 方程式

重力ポテンシャルΦを導入 : $\frac{d\Phi}{dr} = \frac{Gm}{r^2}$

$$(1.2) \rightarrow \frac{dP}{dr} = -\frac{d\Phi}{dr}\rho \qquad (A)$$

*Poisson's*方程式

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{d\Phi}{dr}) = 4\pi G\rho \qquad (B)$$

$$(A) \rightarrow \frac{d\Phi}{dr} = -\gamma K \rho^{\gamma-2}\frac{d\rho}{dr} \qquad (P = K\rho^{\gamma} \equiv K\rho^{1+\frac{1}{n}})$$

$$If \quad \gamma \neq 1 \rightarrow \rho = \left(\frac{-\Phi}{(n+1) \ K}\right)^n$$

$$(B) \rightarrow \frac{d^2\Phi}{dr^2} + \frac{2}{r}\frac{d\Phi}{dr} = 4\pi G\left(\frac{-\Phi}{(n+1) \ K}\right)^n$$

ここで　$z = Ar,$

$$A^2 = \frac{4\pi G}{(n+1)^n K^n}(-\Phi_c)^{n-1} = \frac{4\pi G}{(n+1) \ K}(\rho_c)^{\frac{n-1}{n}}$$

$$w = \frac{\Phi}{\Phi_c} = \left(\frac{\rho}{\rho_c}\right)^{\frac{1}{n}}$$ （添え字cは星の中心での値）

（中心$r=0$で$z=0, \Phi=\Phi_c, \rho=\rho_c, w=1$）

とすると、*Lane–Emden*方程式が得られる。

$$\frac{d^2w}{dr^2} + \frac{2}{z}\frac{dw}{dz} + w^n = 0 \quad or \quad \frac{1}{z^2}\frac{d}{dz}\left(z^2\frac{dw}{dz}\right) + w^n = 0$$

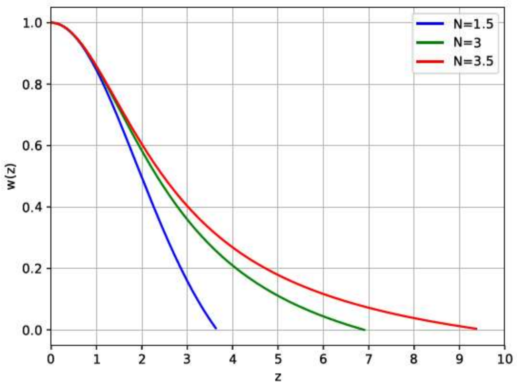
Lane – Emden equation

$$\frac{1}{z^2}\frac{d}{dz}\left(z^2\frac{dw}{dz}\right) + w^n = 0$$

$$M = 4\pi\rho_c R^3\left(-\frac{1}{z}\frac{dw}{dz}\right)_{z=z_n}$$

$$\frac{\bar{\rho}}{\rho_c} = \left(-\frac{3}{z}\frac{dw}{dz}\right)_{z=z_n}$$

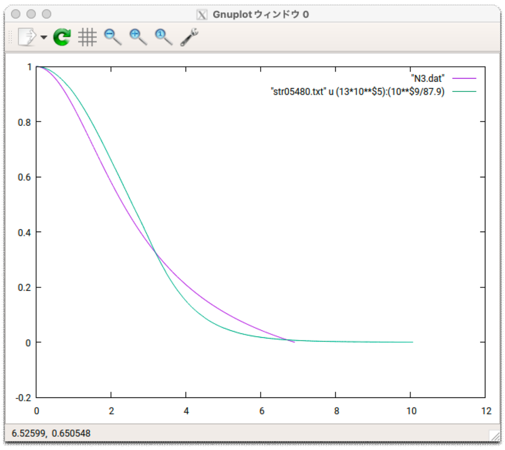
n	z_n	$\left(-z^2\frac{dw}{dz}\right)_{z=z_n}$	$\varrho_c/\bar{\varrho}$
0	2.4494	4.8988	1.0000
1	3.14159	3.14159	3.28987
1.5	3.65375	2.71406	5.99071
2	4.35287	2.41105	11.40254
3	6.89685	2.01824	54.1825
4	14.97155	1.79723	622.408
4.5	31.8365	1.73780	6189.47
5	∞	1.73205	∞



Using python SciPy odeint

N=3 ポリトロープと星の数値計算（1M$_\odot$ ZAMS） の比較

密度 中心で規格化



半径
（星の計算の方は適当にスケール）

恒星の誕生から主系列星になるまで

- 重力によって自己収縮しているガス雲は密度が上がり光学的に厚くなると断熱的に温度が上昇していく
- 中心星へのガスの降着率が減って、accretion time-scale が重力収縮の time-scale, Kelvin-Helmholtz time-scale, より長くなると星は質量ほぼ一定の単独星として進化を始める。
- その後の進化は準静的であると考えて良く、系のエネルギーの進化はビリアル定理（1．6）に従う。
  - $E_{KE} = |E_{GR}|/2$ であり、解放される重力エネルギーの半分が蓄えられ、半分は星の光度として放出される。

Protostars, Proto-planetary disk

イラスト： NASA

Grav. Energy～ -GM^2 / R

Half of the energy is inside and the rest goes outside. (Very Bright but usually hidden by dusts：Herbig Ae/Be star、T Tauri star）

L ~ GM(dM/dt) / R
~ 300 (M / M$_\odot$) (dM/dt$_{-5}$) (R$_\odot$ /R) L$_\odot$

準静的収縮

Quasi-static Contraction

$$E_{GR} = -\int_0^R \frac{Gm\rho}{r} 4\pi r^2 dr = -\frac{3GM^2}{5R} \qquad (\rho = 一定の時)$$

一般に $\quad E_{GR} \approx -\frac{GM^2}{R}$

$$(1.5) \rightarrow \quad P_c \propto \langle P \rangle \approx \frac{GM^2}{4\pi R^4} \qquad (1.8)$$

ρ_cは星の平均密度に比例 $\quad \rho_c \propto \frac{M}{\frac{4}{3}\pi R^3}$

$$\frac{P_c}{\rho_c} \propto \frac{GM}{R} \quad (1.9), \qquad \frac{P_c^3}{\rho_c^4} \propto G^3 M^2 \quad (1.10)$$

理想気体状態方程式 $\quad P_c = \frac{k}{\mu m_u} \rho_c T_c$

$$+(1.10) \rightarrow \frac{T_c^3}{\rho_c} \propto G^3 M^2 \qquad (1.11)$$

(1.11)式 $\Rightarrow M$一定のとき$\underline{\rho_c \propto T_c^3}$

Quasi-static contraction

エントロピー(単位体積あたりの)

$$s = \frac{k}{\mu m_u}\ln(T^{3/2}/\rho) + C_1,$$

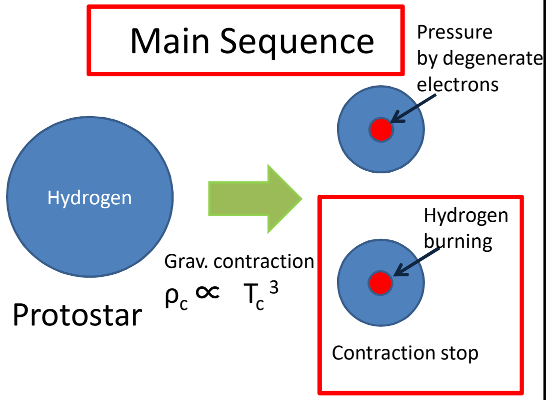
$$+(1.11) \rightarrow$$

$$\left(\frac{\mu m_u}{k}\right)s_c = \ln\left(\frac{M^2}{T_c^{3/2}}\right) + C_2 = \ln\left(\frac{M}{\rho_c^{1/2}}\right) + C_3 \quad (1.12)$$

(1.12)式 $\Rightarrow$ M一定でs_cが減少するとT_c上昇

比熱は負 $C_g = \frac{ds_c}{d\ln T_c} = -1.5\frac{k}{\mu m_\mu} \quad (1.13)$

Main Sequence



- **Maximum Stellar temperature determined by the degenerate pressure (rough estimate)**

(1.6)ビリアル定理 $2E_{KE} + E_{GR} = 0$

$$E_{GR} \approx -\frac{GM^2}{R}, \quad E_{KE} = \frac{3}{2}NkT \rightarrow kT \approx \frac{GMm_u}{3R} \approx Gm_u M^{2/3}\rho^{1/3}$$

M/N

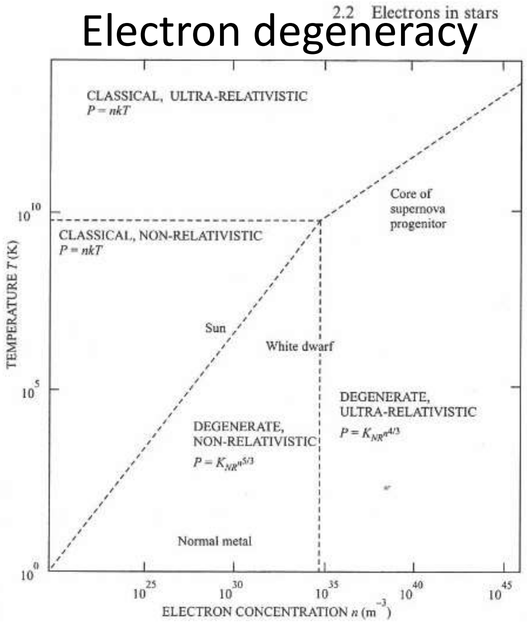
+縮退圧の効く密度：ド ・ブロイ波長

$$\lambda \approx \frac{h}{\sqrt{m_e kT}}, \quad \rho \approx \frac{m_u}{\lambda^3} \approx m_u\frac{(m_e kT)^{3/2}}{h^3}$$

M/R^3 ~ ρ

$$kT_{max} \approx \left[\frac{G^2 m_u^{8/3} m_e}{h^2}\right]M^{4/3} \quad (1.14)$$

Electron degeneracy

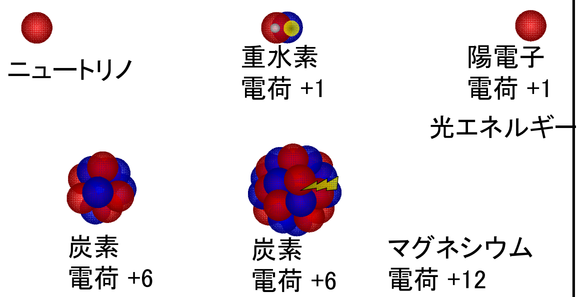


- In a white dwarf, electrons are degenerate but nuclei are not.

Nuclear fusion

➲ 地球上では原子核の電気の反発により天然には核融合は起こらない（加速器や原子炉では可能）

➲ 星の内部は高温 (T > 10^7 K) になると，核融合が起こる。すなわち，異なる元素を作ることができる（Quantum mechanical tunneling effect）



➲ 星は核融合のエネルギーで輝いている（次世代のエネルギー）

28

核融合

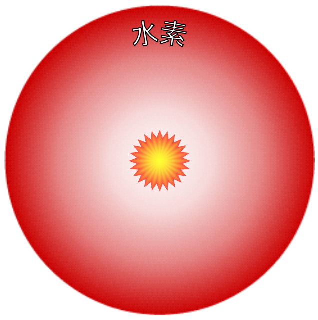
- 量子力学的計算(KH178)
  - クーロン障壁
  - Tunnelling probability
  - 反応断面積、Sファクター

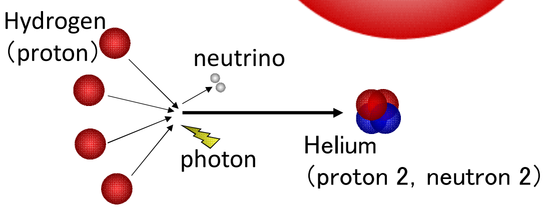
水素燃焼 (KH193)

太陽ニュートリノ (KH338)

Sun is a nuclear fusion reactor

- Tc = 1.5x10^{7} K
- Shining by the fusion energy of hydrogens





- We are observing photons created 1 million years ago and neutrino created 8 min 19 sec ago.

-10

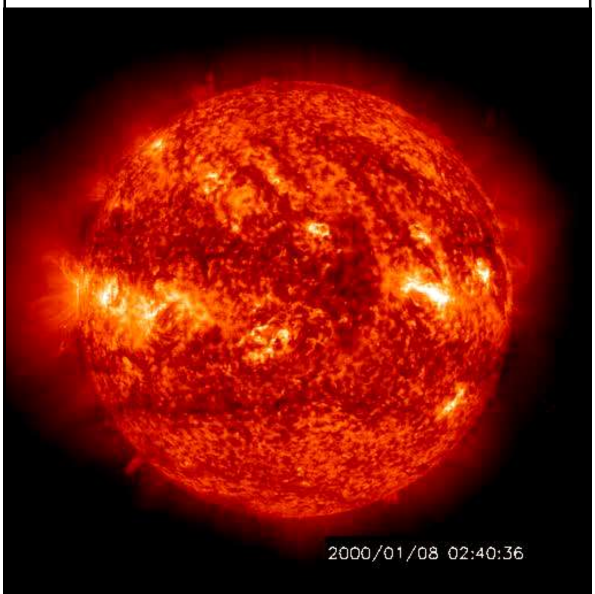
Hydrogen Burning

- (1.14)→ $M < 0.08M_\odot - T_{max} < \sim 10^7$ K Hydrogen can't be ignited.
  ➡ Brown dwarfs (褐色矮星, supported by the degenerate pressure)
- T ~ 1.5x10^{7} K: **pp (proton-proton) chain**

$$p + p \to d + e^+ + \nu_e$$
$$p + d \to {}^3He + \gamma$$
$$1)\ {}^3He + {}^3He \to {}^4He + p + p \quad (pp1)$$
$$2)\ {}^3He + {}^4He \to {}^7Be + \gamma$$
$$2a)\ e^- + {}^7Be \to {}^7Li + \nu_e$$
$$p + {}^7Li \to {}^4He + {}^4He \quad (pp2)$$
$$2b)\ p + {}^7Be \to {}^8B + \gamma$$
$${}^8B \to {}^8Be^* + e^+ + \nu_e$$
$${}^8Be^* \to {}^4He + {}^4He \quad (pp3)$$

$$\varepsilon_{pp} \sim 2.36\times10^6 X_H^2 \rho T_6^{-2/3} \times \exp(-33.8\, T_6^{-1/3}) \ \text{erg g}^{-1}\text{sec}^{-1}$$

太陽



32

Solar neutrino

- **pp (proton-proton) chain**

$p + p \rightarrow d + e^+ + \nu_e$

$p + d \rightarrow {}^3He + \gamma$

1) ${}^3He + {}^3He \rightarrow {}^4He + p + p$ (pp1)

2) ${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

2a) $e^- + {}^7Be \rightarrow {}^7Li + \nu_e$

$p + {}^7Li \rightarrow {}^4He + {}^4He$ (pp2)

2b) $p + {}^7Be \rightarrow {}^8B + \gamma$

${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$

${}^8Be^* \rightarrow {}^4He + {}^4He$ (pp3)

Process	Neutrino flux (10^{14} m^{-2} s^{-1})	Maximum neutrino energy (MeV)
$p + p \rightarrow d + e^+ + \nu_e$	6.0 (1 ± 0.02)	0.420
$e^- + {}^7Be \rightarrow {}^7Li + \nu_e$	0.47 (1 ± 0.15)	0.861
${}^8B \rightarrow {}^8Be + e^+ + \nu_e$	5.8 × 10^{-4} (1 ± 0.37)	15
${}^{13}N \rightarrow {}^{13}C + e^+ + \nu_e$	0.06 (1 ± 0.50)	1.199
${}^{15}O \rightarrow {}^{15}N + e^+ + \nu_e$	0.05 (1 ± 0.58)	1.732

$\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e^+$ (detectable only above 0.81MeV)

Predicted 7.9±2.6 SNU, Observed 2.5±0.4 SNU

Kamioka $\nu_e + {}^2H \rightarrow p + p + e^-$, $\nu_x + {}^2H \rightarrow n + p + \nu_x$

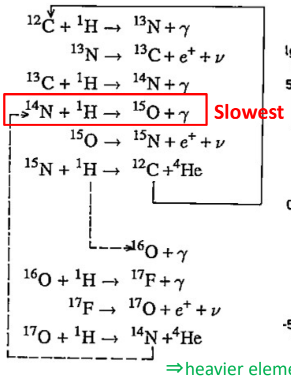
Result ν_e changes to ν_{mu} & ν_{tau}

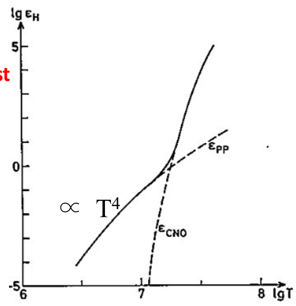
Discovery of the neutrino oscillation (& neutrino mass)

Hydrogen burning

- $M > 1\ M_\odot$, at $T \sim 2\times10^7$ K

CNO cycle





⇒heavier elements can be produced a little

$\varepsilon_{CNO} \sim 8.67\times10^{27} X_{CNO} X_H \rho T_6^{-2/3}$

$\times \exp(-152.3\ T_6^{-1/3})$ erg g^{-1}sec^{-1}

$\propto \rho T^{16}$ at $T \sim 2\times10^7$ K

${}^{12}C : {}^{13}C : {}^{14}N : {}^{15}N = 4 : 1 : 95 : 0.004$

Stellar evolution calculation

- Basic equation : 1 D-Spherical
  - continuity $\frac{dm}{dr} = 4\pi r^2 \rho$ (1.1)
  - hydro static $\frac{dP}{dr} = -\frac{Gm\rho}{r^2}$ (1.2)
  - Equation of state (EOS) P(ρ、T)

Ideal gas $P = \frac{k}{\mu m_u}\rho T$ (1.3)

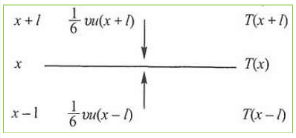
Completely degenerate gas $P \propto \rho^{5/3}(non-relativistic)$

$P \propto \rho^{4/3}(relativistic)$

- Other equation
  - Energy generation and chemical evolution
    - After the main-sequence, stars are sustained with the nuclear fusion energy
    - By solving nuclear reaction networks, one can calculates energy generation and chemical evolution at the same time (later for detail)
- Below, following equations are explained
  - Energy transfer equation (radiation & convection)
  - Energy conservation equation

Energy transfer in a star

- **Radiation (rondom motion of photons)**
  - u(x) : thermal energy per unit volume at *x*
  - Assuming that 1/6 of the photons moves toward *x* direction, and letting the mean free path *l*



Heat flux density : $j(x) =$

$\frac{1}{6}vu(x-l) - \frac{1}{6}vu(x+l)$

$= -\frac{1}{3}vl\frac{du}{dx} = -\frac{1}{3}vl\frac{du}{dT}\frac{dT}{dx}$

$= -K\frac{dT}{dx}$ $\quad (K = \frac{1}{3}vlC)$

Heat conduction coefficient

Heat capacity per unit volume

Radiative diffusion :

$u_r = aT^4, C_r = 4aT^3, K_r \approx \frac{4}{3}claT^3$

$l = (\rho\kappa)^{-1}$ $\quad (\kappa : \text{opacity})$

$\rightarrow j(x) = -\frac{4ac}{3}\frac{T^3}{\rho\kappa}\frac{dT}{dx}$

Energy transfer in a star

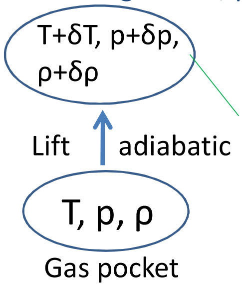
- When radiative diffusion dominates, energy flow rate *L(r)* is

$$L(r) = 4\pi r^2 j(r) = -4\pi r^2 \frac{4ac}{3}\frac{T^3}{\rho\kappa}\frac{dT}{dr}$$

$$\rightarrow \left[\frac{dT}{dr}\right]_{rad} = -\frac{3\rho\kappa}{4acT^3}\frac{L}{4\pi r^2} \quad (1.15)$$

- Transfer by convection

Surroundings : T+ΔT, p+Δp, ρ+Δρ



Keep moving up, If the lifted gas pocket has lower density than the surroundings.

Condition for convection
δρ < Δρ (|δρ| > |Δρ|)

Energy transfer inside a star

- Transfer by convection

$$\text{adiabatic} \rightarrow p \propto \rho^{\gamma} \rightarrow \frac{\delta\rho}{\rho} = \frac{1}{\gamma}\frac{\delta p}{p}$$

$$\text{ideal gas } \rho \propto \frac{p}{T} \rightarrow \frac{\Delta\rho}{\rho} = \frac{\Delta p}{p} - \frac{\Delta T}{T}$$

Convective condition
δρ < Δρ (|δρ| > |Δρ|)

Pressure inside the gas pocket quickly is balanced with outside: δp = Δp

$$\rightarrow \frac{1}{\gamma}\frac{\delta p}{p} < \frac{\Delta p}{p} - \frac{\Delta T}{T} \rightarrow \frac{\Delta T}{T} < \frac{\gamma - 1}{\gamma}\frac{\Delta p}{p}$$

$$\rightarrow \frac{dT}{dr} < \frac{\gamma - 1}{\gamma}\frac{T}{p}\frac{dp}{dr} \equiv \left[\frac{dT}{dr}\right]_{crit} \quad (\text{Note}: \frac{dT}{dr} < 0)$$

Summary of Energy transfer equations

- Transfer efficiency by convection is very high, so in the convective region: $\frac{dT}{dr} = \left[\frac{dT}{dr}\right]_{crit}$

$$\text{*if } \left|\frac{dT}{dr}\right| < \left|\frac{dT}{dr}\right|_{crit} \quad (radiative)$$

$$\frac{dT}{dr} = -\frac{3\rho\kappa}{4acT^3}\frac{L}{4\pi r^2} \quad (1.15)$$

$$\text{*if } \left|\frac{dT}{dr}\right| > \left|\frac{dT}{dr}\right|_{crit} \quad (convective)$$

$$\frac{dT}{dr} = \frac{\gamma - 1}{\gamma}\frac{T}{p}\frac{dp}{dr}\left(\equiv \left[\frac{dT}{dr}\right]_{crit}\right)$$

$$= -\frac{\gamma - 1}{\gamma}\frac{T}{p}\frac{Gm\rho}{r^2} \quad (1.16)$$

$$(\text{equation for hydrostatic } \frac{dp}{dr} = -\frac{Gm\rho}{r^2})$$

Energy transfer equation (supplement)

$$\left(\frac{dT}{dr}\right)_{crit} = \frac{\gamma - 1}{\gamma}\frac{T}{P}\frac{dP}{dr}$$

$$\rightarrow \frac{P}{T}\left(\frac{dT}{dP}\right)_{crit} = \left(\frac{d\ln T}{d\ln P}\right)_{crit} = \frac{\gamma - 1}{\gamma}$$

$$\text{define } \frac{d\ln T}{d\ln P} \equiv \nabla, \text{ then } \nabla_{crit} = \frac{\gamma - 1}{\gamma}$$

$$(1.15 \text{ when radiative}) \rightarrow \nabla_{rad} = \frac{P}{T}\frac{dr}{dP}\frac{dT}{dr} = \frac{3\kappa LP}{16\pi acT^4 Gm}$$

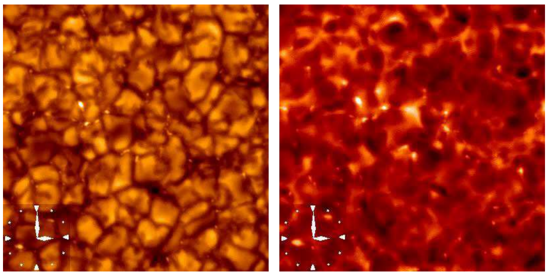
therefore, energy transfer eqation is
when radaiative $(\nabla < \nabla_{crit}) : \nabla = \nabla_{rad}$
when convective : $\nabla = \nabla_{crit}$

* About Le doux criterion

Example of the convective regions

- Central part of massive stars & Outside of the shell burning region:
  nuclear burning is strong (large|dT/dr|)
- Regions where dissociation of molecule and ionization occurs (γ = ~1):
  e.g. ~$0.2R_\odot$ below of photosphere of the Sun

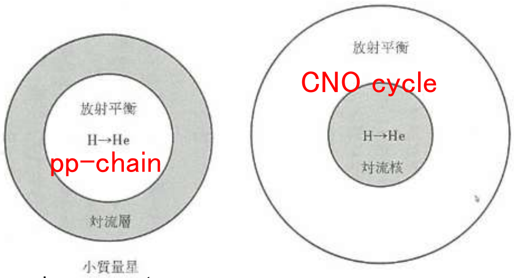
Solar optical telescope satellite ひので「Hinode」



光球　　彩層

Convective and radiative layers in main-sequence stars

M ~ 0.1 -1 $M_\odot$　　M > 1 $M_\odot$



小質量星
Low mass stars

Outer layer is convective

大中質量星
Intermediate and massive stars

Central part is convective

エネルギー保存の式 (KH30)

- 完全に静的(stationary)な場合, エネルギー生成率εを持つ厚さdrの球殻を通過するときのenergy flow rate (luminosity) の変化量は
  dL = $4\pi r^2 \rho\, \varepsilon\, dr = \varepsilon\, dm$　又は
- 一般に

$$dq = \left(\varepsilon - \frac{\partial L}{\partial m}\right)dt = Tds \rightarrow T\frac{\partial s}{\partial t} = \varepsilon - \frac{\partial L}{\partial m}$$

$$\varepsilon = \varepsilon_n - \varepsilon_\nu, \quad \varepsilon_g \equiv -T\frac{\partial s}{\partial t}$$ とすると

ε_n : nuclear energy generation rate

ε_ν : neutrino energy loss rate

$$\frac{\partial L}{\partial m} = \varepsilon_n - \varepsilon_\nu + \varepsilon_g$$

(1.17 : エネルギー保存の式)

Energy conservation

Modifying ε_g into a more convenient form (using the first law of thermodynamics)

$$Tds = du + Pdv = \left(\frac{\partial u}{\partial T}\right)_v dT + \left[P + \left(\frac{\partial u}{\partial v}\right)_T\right]dv$$

$$= c_v dT - \frac{P\delta}{\rho^2 \alpha} d\rho \quad (v = 1/\rho : \text{specific volume},$$

c_v : specific heat at constant volume)

$$\alpha \equiv \left(\frac{\partial \ln \rho}{\partial \ln P}\right)_T, \quad \delta \equiv -\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_P$$

$$\varepsilon_g = -T\left(\frac{\partial s}{\partial t}\right) = -c_v \frac{\partial T}{\partial t} + \frac{P\delta}{\rho^2 \alpha}\frac{\partial \rho}{\partial t}$$

$$= -c_p T\left(\frac{1}{T}\frac{\partial T}{\partial t} - \frac{\nabla_{ad}}{P}\frac{\partial P}{\partial t}\right) \quad (1.18)$$

c_p : specific heat at constant pressure

$$\nabla_{ad} \equiv \left(\frac{\partial \ln T}{\partial \ln P}\right)_s = \frac{P\delta}{T\rho\, c_p}$$

: adiabatic temperature gradient (1.19)

星の進化計算
Full set of equations

- Rewrite equations as a function of *m*

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \varrho},$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2}, \quad \text{(with acceleration term)}$$

$$\frac{\partial L}{\partial m} = \varepsilon_n - \varepsilon_\nu - c_P\frac{\partial T}{\partial t} + \frac{\delta}{\varrho}\frac{\partial P}{\partial t}$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla, \quad \left(\nabla = \nabla_{rad} = \frac{3\kappa L P}{16\pi a c T^4 G m}, \nabla = \nabla_{convec}\right)$$

$$\frac{\partial X_i}{\partial t} = \frac{m_i}{\varrho}\left(\sum_j r_{ji} - \sum_k r_{ik}\right)$$

(境界条件 KH p.93~)
↓

- Usually solved by the Henyey method
  - Non-liner boundary condition problem (boundary conditions depend on time implicitly)
  - Define δy as the difference between the temporal and true solution. Then linearize the equation for δy to solve
  - Repeat (iterate) this procedure until δy becomes sufficiently small

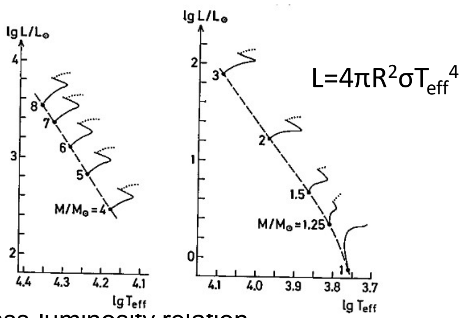
Time scales

- Nuclear Timescale $\tau_n = E_n/L$ (KH p.35)
  (E_n : nuclear energy)
  ($\tau_n \sim 10^{11}$ years for the Sun)
- Hydrostatic timescale
  $\tau_{hydr} \approx (R^3/GM)^{1/2} \approx ½*(G<\rho>)^{-1/2}$ (KH p.14)
  ($\tau_{hydr} \sim$ 27 min for the Sun,
  ~18 days for a Red giant)
- Kelvin-Helmholtz timescale
  $\tau_{KH} = E_g / L \approx (GM^2/2RL)$ (KH p.22)
  (E_g : gravitational energy)
  ($\tau_{KH} \sim 10^7$ years for the Sun)

通常 $\tau_n \gg \tau_{KH} \gg \tau_{hydr}$

Mass, Radius of main-sequence star and HR diagram

- Larger L is required for a larger M to sustain the star. However, Tc is almost unchanged because $\varepsilon_{CNO} \propto T^{16}$.
  (1.9) → $R \propto M/T_c \propto M$
- Hertzsprung-Russell diagram



Mass-luminosity relation
$L \propto \sim M^3$ ($M \gtrsim 1M_\odot$) → $T_{eff} \propto M^{1/4}$
lifetime $\propto M/L \sim M^{-2}$ (more massive stars have shorter lifetime)

(More precise) Observational fit (主系列星)

- $R \propto M^{0.6}$ くらい
- Mass-luminosity relation
  $L \propto M^{\alpha}$
  α＝2.3 ($M < 0.43M_\odot$)
  = 4 ($0.43M_\odot < M < 2 M_\odot$)
  = 3.5 ($2M_\odot < M < 20 M_\odot$)
  = 1 ($20M_\odot < M$)