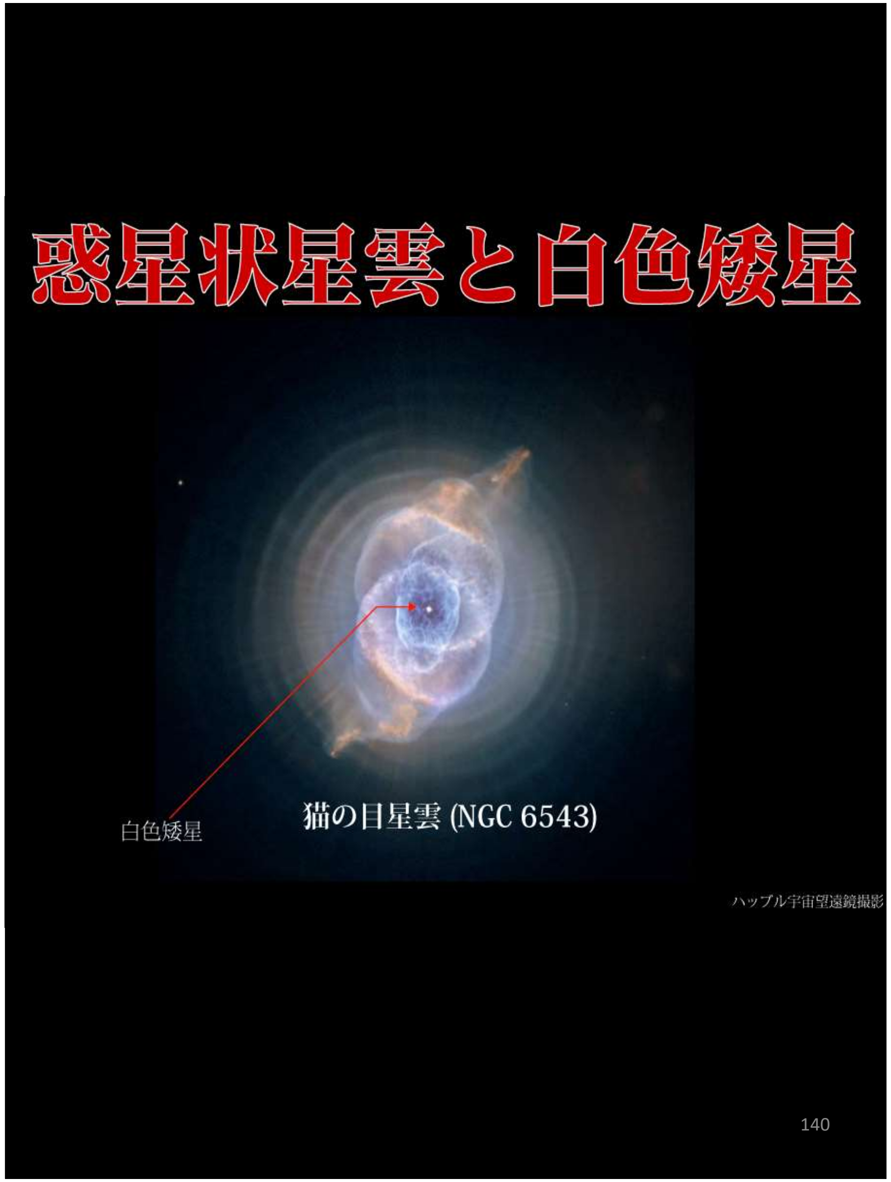


# 惑星状星雲と白色矮星

猫の目星雲 (NGC 6543)

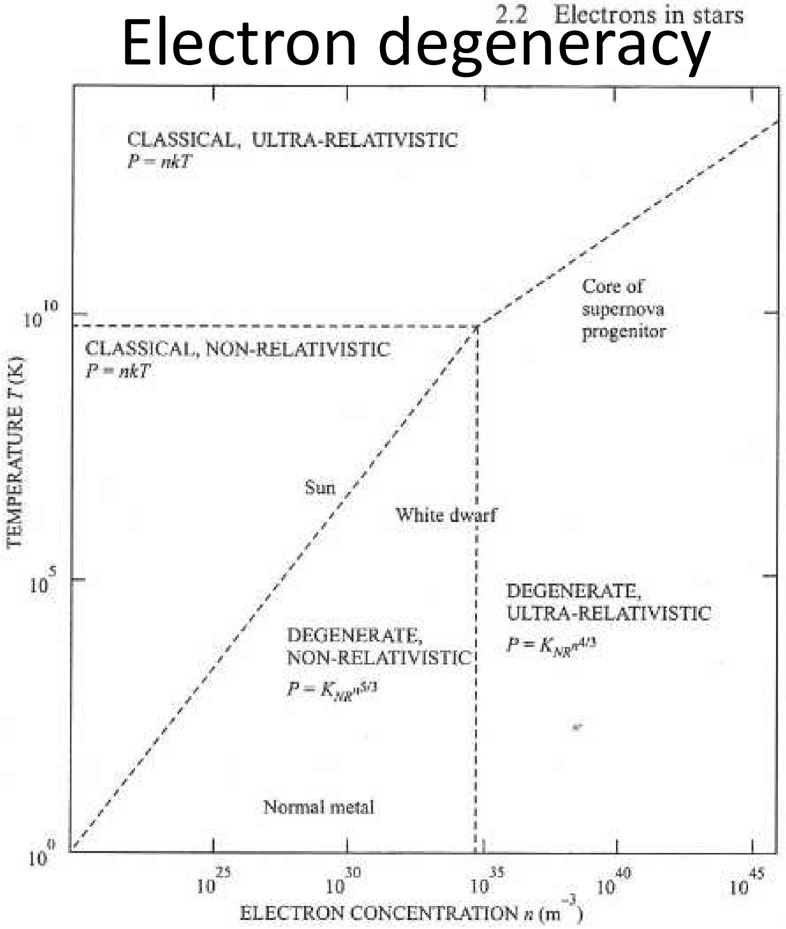
ハッブル宇宙望遠鏡撮影

# 白色矮星

- 0.8 $M_{\odot}$より軽い星は寿命が宇宙年齢よりも長いためまだ主系列にいる
- 0.8 $M_{\odot} < M < 8\ M_{\odot}$の星はHeが燃え尽きた後、炭素に点火しないのでCO核を持ったAGB星となる。 (e.g. Umeda et al. 1999)
- AGB星は質量放出が激しいため、その後CO核の質量はほとんど増えずにその大きさのCO白色矮星となる。
- 同様にM ~ 8 -10$M_{\odot}$の星は炭素が燃えてO-Ne-Mgを主成分とする白色矮星となる

- 白色矮星の典型的質量はM~0.6 $M_{\odot}$で半径は1000 k m程度。密度は$10^6$ g $cm^{-3}$以上。
- 核燃焼が起きないため冷える一方であるが、冷却のタイムスケールが非常に長いため、冷却曲線と観測を比べることにより宇宙年齢を制限する一つの手段として使われている(た )

# Electron degeneracy



- In a white dwarf, electrons are degenerate but nuclei are not.
- Next: Chandrasekhar limit

# Polytrope & White dwarfs

- Non-relativistic, complete degenerate

$$P = K_{NR} n_e^{5/3} = K_{NR} \left( \frac{Y_e \rho}{m_u} \right)^{5/3}$$

$Y_e$ : electron number per nucleon

$$K_{NR} = \frac{h^2}{5 m_e} \left( \frac{3}{8\pi} \right)^{2/3}$$

$$1 + \frac{1}{n} = \frac{5}{3} \rightarrow n = \frac{3}{2} \text{ Polytrope}$$

- Ultra relativistic, complete degenerate

$$P = K_{UR} n_e^{4/3}, K_{UR} = \frac{hc}{4} \left( \frac{3}{8\pi} \right)^{1/3}$$

$n = 3$ Polytrope

## Polytrope & White dwarsf

- Completely degenerate electron gas :
  *n* changes from 1.5 to 3.0 as the density increases

In general, for a polytropic star,

$$\text{Stellar radius}: \ R = \frac{z_n}{A} \propto A^{-1} \propto \rho_c^{\frac{1-n}{2n}}$$

$$\text{Mass}: \ \mathrm{M} = 4\pi\rho_c R^3 \left( -\frac{w'}{z} \right)_{z=z_n} \propto \rho_c^{\frac{3-n}{2n}}$$

$$\rightarrow R \propto M^{\frac{1-n}{3-n}}$$

When 1<*n*<3, *R* becomes smaller as *M* increases.

For the limit of *n* →3（relativistic limit）, *R* → 0.

## Polytrope & White dwarfs

$$K = c_1 A^{-2} G \rho_c^{\frac{n-1}{n}} \quad (\mathrm{KH\,19.9}),$$

$c_i$ : non dimensional constants

$$\& \ \rho_c \propto \overline{\rho} \propto MR^{-3} = M\left(\frac{A}{z_n}\right)^3$$

$$\rightarrow A = c_2 \rho_c^{1/3} M^{-1/3}$$

$$\rightarrow K = c_3 M^{2/3} G \rho_c^{1-\frac{1}{n}-\frac{2}{3}} = c_3 M^{2/3} G \rho_c^{\frac{1}{3}-\frac{1}{n}},$$

$$\text{also}: P_c = K \rho_c^{1+\frac{1}{n}} = c_3 GM^{2/3} G \rho_c^{\frac{4}{3}}$$

$$P_c = 0.48 \ \ GM^{2/3} G \rho_c^{\frac{4}{3}} \qquad (n = \frac{3}{2}) \quad (1.21)$$

$$= 0.36 \ \ GM^{2/3} G \rho_c^{\frac{4}{3}} \qquad (n = 3) \quad (1.22)$$

## Polytrope & White dwarfs

For the limit of ultra-relativistic (or high density), EOS is $P_c \propto \rho_c^{4/3}$. Using this and (1.22) we find mass approaches to

$$M = 1.46\left(\frac{Y_e}{0.5}\right)^2 M_\Theta \equiv M_{Ch},$$

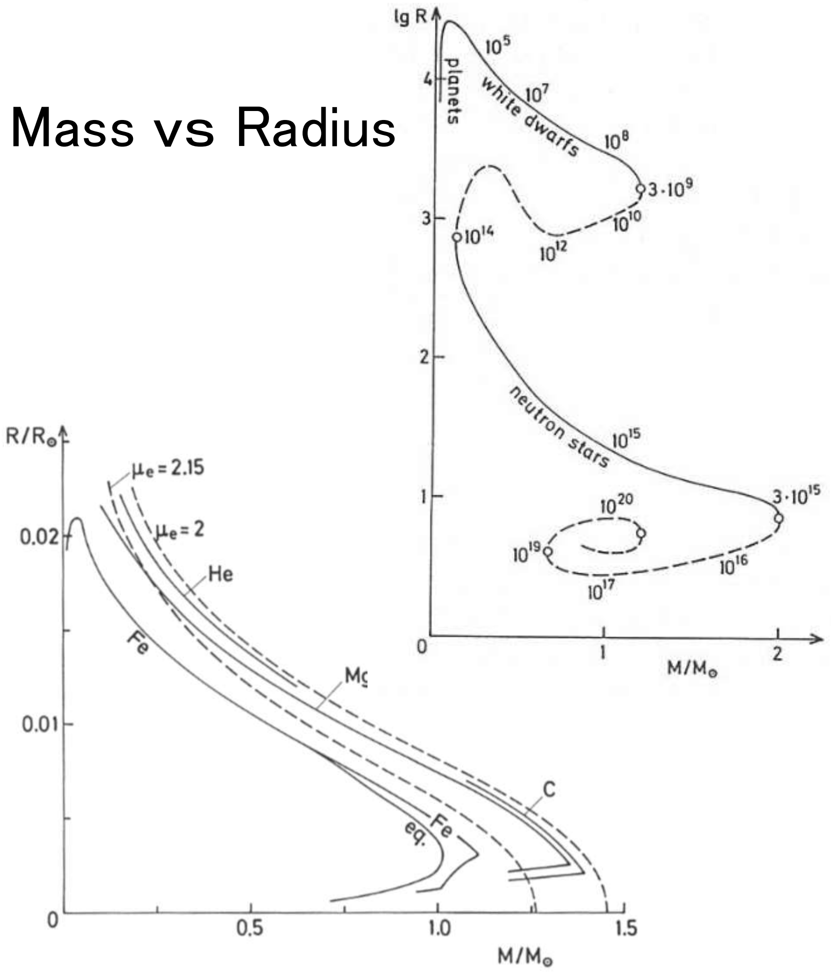
the Chandrasekhar mass.

Or when $M \to M_{Ch}$, it becomes $n \to 3$

$$R \propto \lim_{n\to 3} M^{\frac{1-n}{3-n}} = \lim_{n\to 3} M_{Ch}^{\frac{1-n}{3-n}} \to 0, \text{ and}$$

$\rho \to \infty.$

## Mass vs Radius



# 近接連星系の進化

- 連星系の星の距離が非常に近いと2つの星の間でガスの移動が起きる。質量移動の時期とスピードは、連星系の大きさと2つの星の質量比で変わる。
- それぞれの星の重力圏の境界は臨界ロッシュ・ローブと呼ばれ、それらは一点 $L_1$でつながっている。
- 質量の大きい星ほど進化が速いので、先に大きい星が巨星になって臨界ロッシュ・ローブを超える。超えた部分のガスは $L_1$点の近くを通り小さい質量の星に流れ込む。

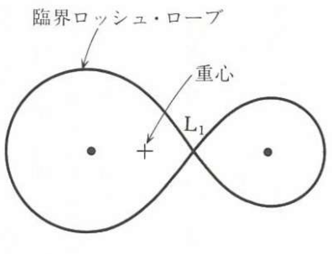


図 1　質量比が1対0.4の連星系における臨界ロッシュ・ローブ

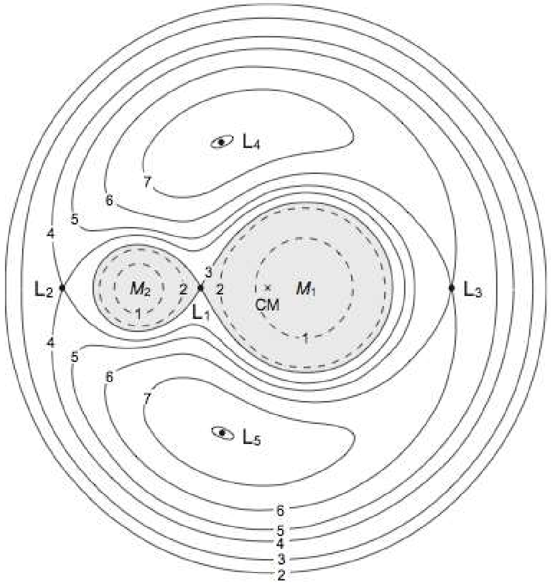


Figure 13.3: Gravitational potential contours for a binary system and the Lagrange points $L_n$. Dashed contours lie inside the Roche lobes (indicated in gray) and CM denotes the location of the center of mass. Adapted from reference [?].

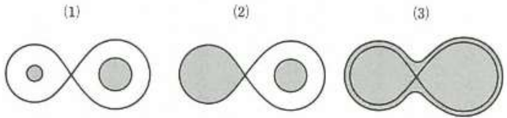
A contour plot corresponding to the potential

$$\Phi_{\mathrm{R}}(\boldsymbol{r}) = -\frac{GM_1}{|\boldsymbol{r}-\boldsymbol{r}_1|} - \frac{GM_2}{|\boldsymbol{r}-\boldsymbol{r}_2|} - \frac{1}{2}(\boldsymbol{\omega}\times\boldsymbol{r})^2,$$

# Evolution of close binary systems

- When total mass and angular momentum is conserved, the distance between two stars is minimum when two stars have equal mass.
  - ➡ mass transfer rate is accelerated

When two stars are very close :

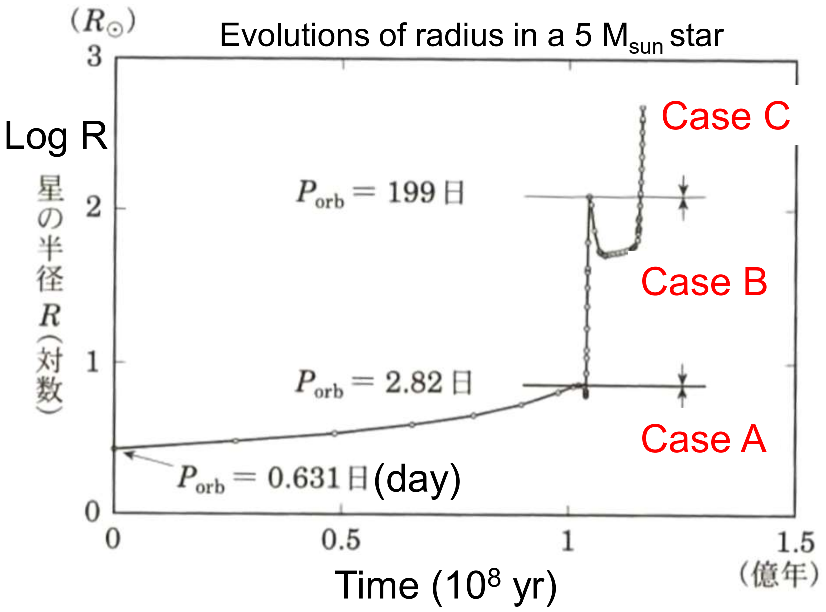


Some patterns of CBS

図 4 近接連星の3つの形態

- Principal star can fill its Roche lobe before the star become a red giant (2) (Case A), then
- transferred mass generates heat. If mass transfer rate is sufficiently large, the companion star also expand, and they may form a common envelope (3).
- After some time, when the evolved star mass becomes smaller the companion, the distance between two stars increases. Then two stars can be detached again. Mass transfer rate decreases and steady mass transfer can occur as (2).
- Depending on the separation and mass ratio, other type of evolution could be possible.

# Cases of binary mass transfer



Case A: A primary star fills the Roche lobe during the main sequence.

time scale: thermal (KH) time scale ~ $GM^2/(RL) = T_{th}$

example: M = 2Msun, $T_{th=}$ $10^7$ yr, dM/dt ~ $10^{-7}$ Msun/yr

M = 5Msun, $T_{th=}$ $10^6$ yr, dM/dt ~ $10^{-5}$ Msun/yr

Case B: A primary star fills the Roche lobe when it becomes a red giant.

time scale:
Convective envelope has large entropy, thus it can expand quickly when the outer most layer lose the mass with the dynamical time scale $T_{dyn} \sim R/v_s \sim (R^3/GM)^{1/2}$
This is much faster than case A.
For example, if R=100 Rsun, $T_{dyn}$ is only 10 days.

Case C: A primary star fills the Roche lobe when it becomes an AGB star.
Time scale is fast as the Case C. Actual transfer rate depends on the masses of stars and separation distances.

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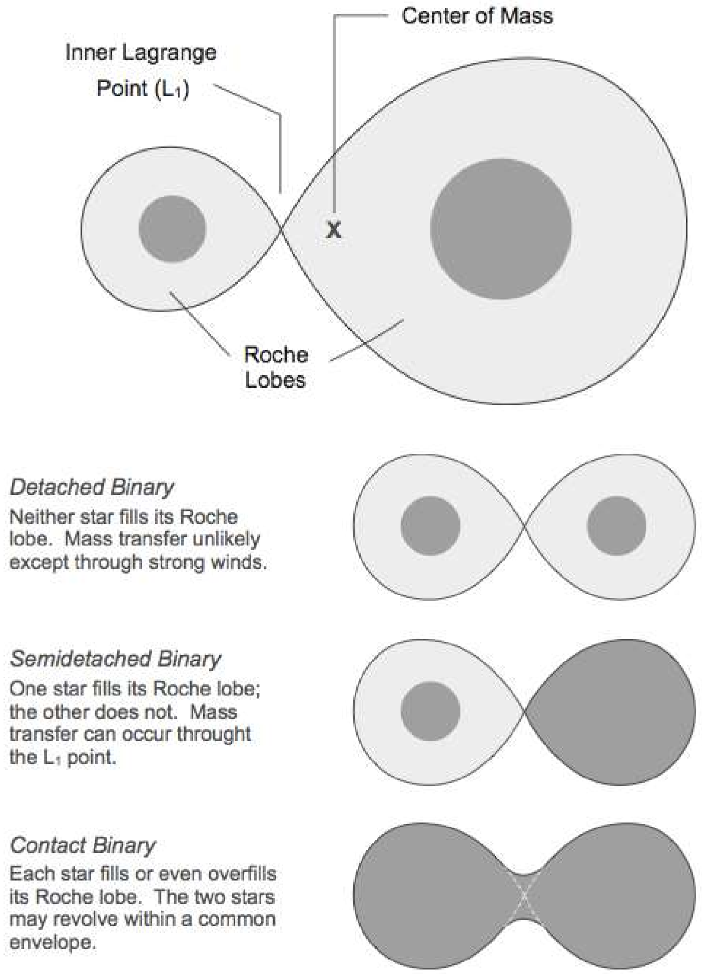


Neither star fills its Roche lobe. Mass transfer unlikely except through strong winds.

One star fills its Roche lobe; the other does not. Mass transfer can occur throught the $L_1$ point.

Each star fills or even overfills its Roche lobe. The two stars may revolve within a common envelope.

//eagle.phys.utk.edu/guidry/astro411/lectures/lecture_ch13.pdf

# 2つの星がもう少し遠い場合の進化

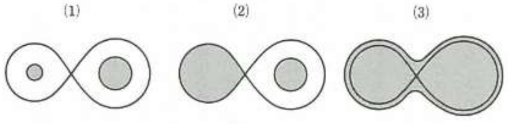


図 4 近接連星の3つの形態

- 片方が赤色巨星になるまで質量移動が起きない場合
- 質量移動が始まるとき、大きい方の星の外層は対流となっている。 表面が対流の星は質量を表面から剥すと膨張する性質がある。
- 質量が移動して質量差が減ると距離が縮む効果もあり、非常に速い質量移動がおき、暴走する。共通外層となったあとも更に膨張し、ガスが連星系から逃げて、進化の進んだ星の外層がほぼ全て失われる。外層が失われると急激に半径が減少し、最後には（1）のような状態になる。
- 外層を失った星はヘリウム星（もしくはCO星）として中心部の進化が進む。中心部の進化は外層のあるなしに殆ど影響されない。

---

# Binary system with a WD

- それぞれの星の質量や距離に応じて、矮新星、新星、Ia型超新星となる。
- 白色矮星の相手の星が進化し、臨界ロッシュ・ローブを超えると白色矮星への物質の降着が始まる。白色矮星は半径がすごく小さいため、ガスは直接降着せずに、白色矮星の周りに回転した降着円盤を形成する。
- ガスはすこしづつ角運動量を失い中心星に近づいていくが、そのときに重力エネルギーを解放し、降着円盤は温められる。

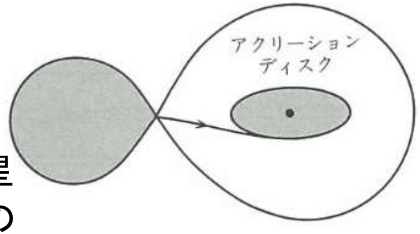


図 5 アクリーションディスク

- 降着円盤の明るさはガスの流入率が一定でも円盤の状態が急に変化して明るさを変える。この変化に伴う星の増光が**矮新星**である。（数日から数年の周期。）

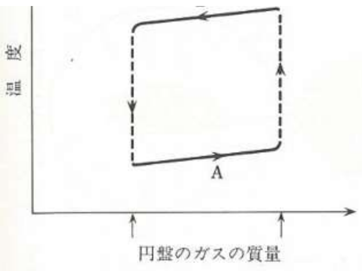
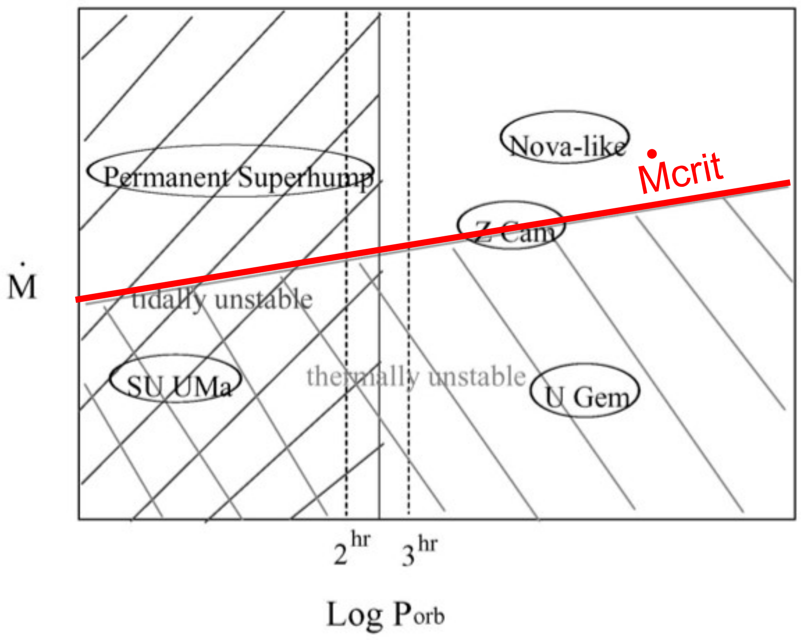


図 6 アクリーションディスクの質量と温度の関係の説明図。横軸の2つの矢印は臨界質量を表わす。破線の部分は変化が速く起こる。

# TTI model (Osaki 2005)



- Smaller $P_{orb}$ means smaller distance
- $\dot{M}_{crit} = 4.3 \times 10^{-9} (P_{orb}/4hr)\ M_{sun}/yr$

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# Binary system of a star and WD：新星 (Nova)

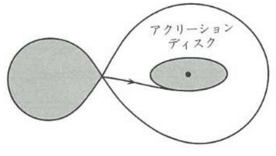


図 5 アクリーションディスク

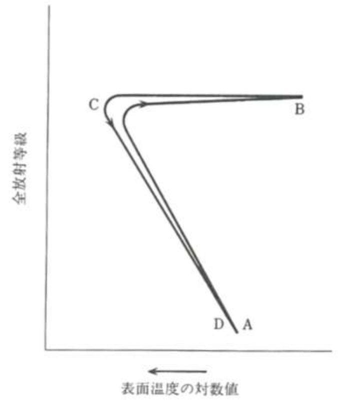


図 8 新星爆発の際の HR 図上の変化の説明図。A 点が爆発発生時点の位置である。

- •白色矮星の表面に水素が
  降り積もり、温度上昇
- •水素燃焼が起きる
- •電子が部分的に縮退しており
  層が薄い⇒核反応は暴走（水素燃焼フラッシュ）

- •A－B:外層が膨張して表面温度下がり、ガス放出
- •B:　表面温度最も低い＝可視光は極大
- •B-C: 堆積した水素のほとんどがHeになるか、
  吹き飛ばされる
- •C-D: 水素がなくなり燃焼終了、再び暗くなる

## Nova and Type Ia supernova

•SN Ia: In a binary system, when a WD gets accretion and reaches the $M_{ch}$, C at center burns explosively. Then whole star is disrupted and becomes a SN Ia.

•After a nova, whether the WD mass increase or not ?： (Now we know 'No'.

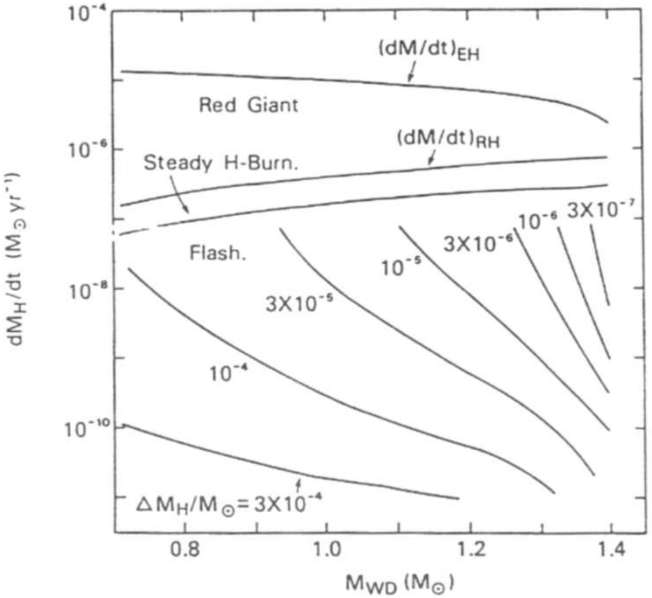
•Anoth

•Wind t

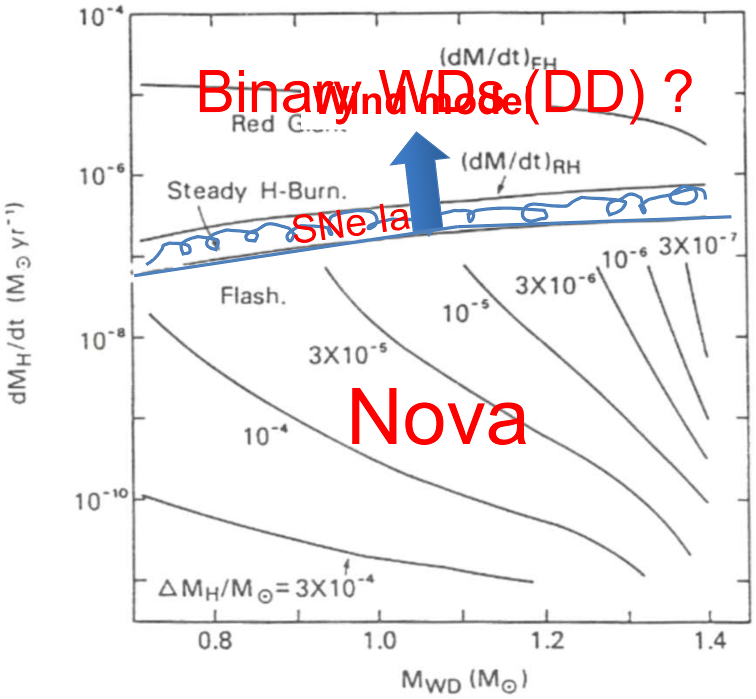
（Kato &

$M_{crit}$ car

•Pathwa



## SD +Wind model



Previously it was shown that steady H-shell burning for the CO core growth Is possible only for a narrow region.

➡ too few SNe Ia from the SD model?

# Critical accretion rate

$$\dot{M}_{cr} = 5.3\times10^{-7}\frac{1.7-X}{X}(M_{WD}-0.4)$$

$$\dot{M}_{He} = \eta_H\left|\dot{M}_2\right|$$ (He core growth rate)

Mass accretion rate onto the star

$$\begin{aligned}
\eta_H &= \dot{M}_{cr}/\left|\dot{M}_2\right|, && \left|\dot{M}_2\right| > \dot{M}_{cr} \\
&= 1, && \dot{M}_{cr} > \left|\dot{M}_2\right| > \frac{1}{8}\dot{M}_{cr} \\
&= 0, && \left|\dot{M}_2\right| < \frac{1}{8}\dot{M}_{cr}
\end{aligned}$$

$$\dot{M}_{WD} = \eta_{He}\dot{M}_{He} = \eta_{He}\eta_H\left|\dot{M}_2\right|$$ (WD growth rate)

$\eta_{He}$ : Mass accumulation efficiency for helium-shell flashes (Kato & Hachisu 2004)

# A pathway to SNe Ia: MS+WD system

(Hachisu, Kato, Nomoto 1999)

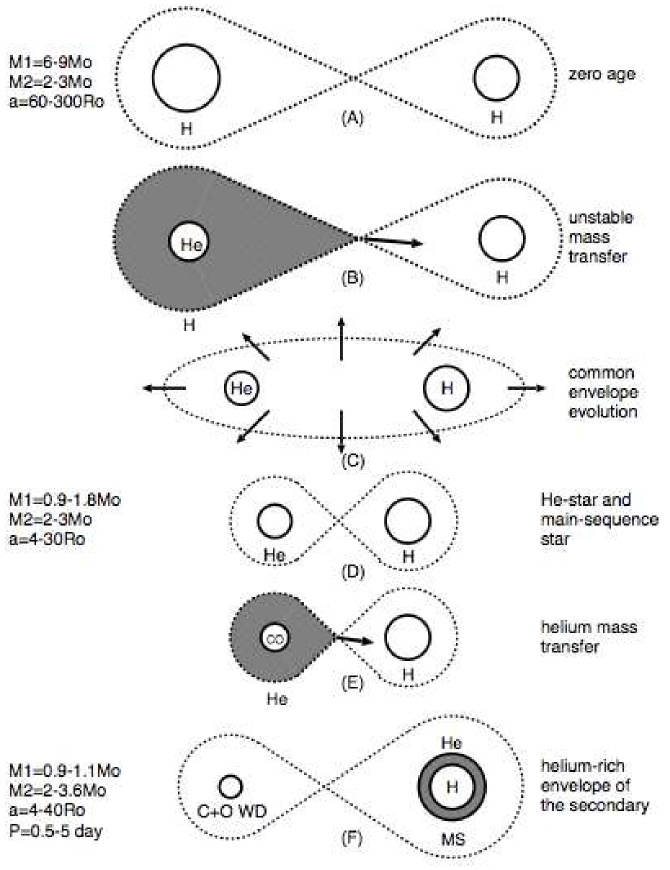


FIG. 1.—Early evolutionary path through the common envelope evolution to the helium matter transfer

# A pathway to SNe Ia:
# MS+WD system (continue)

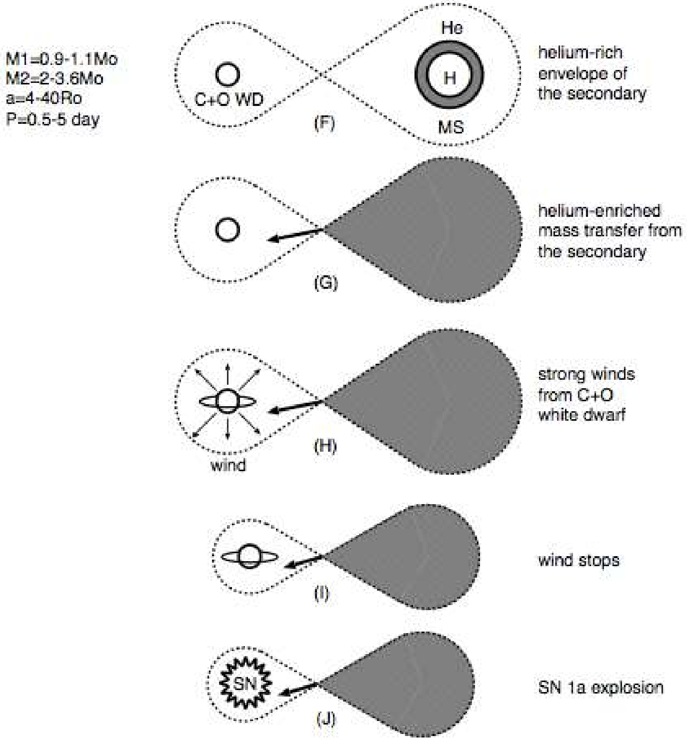
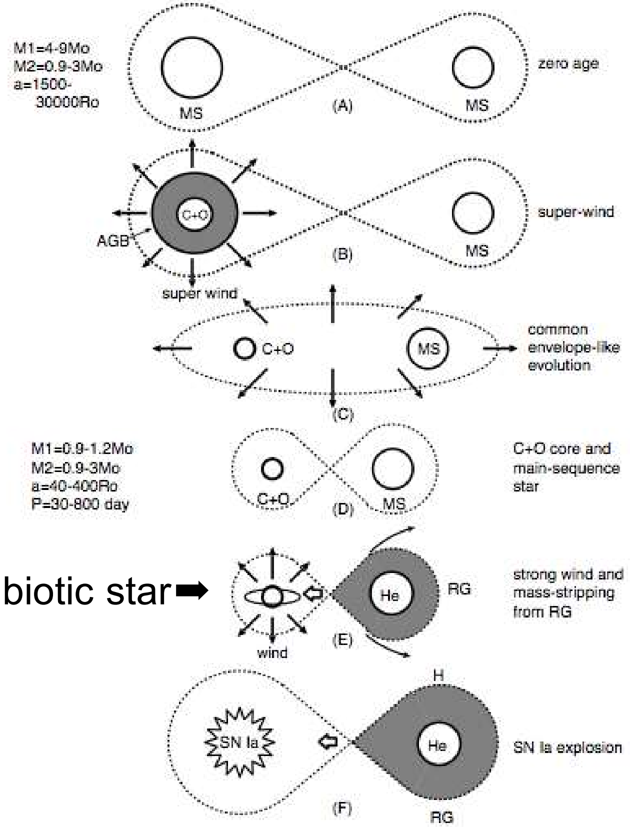


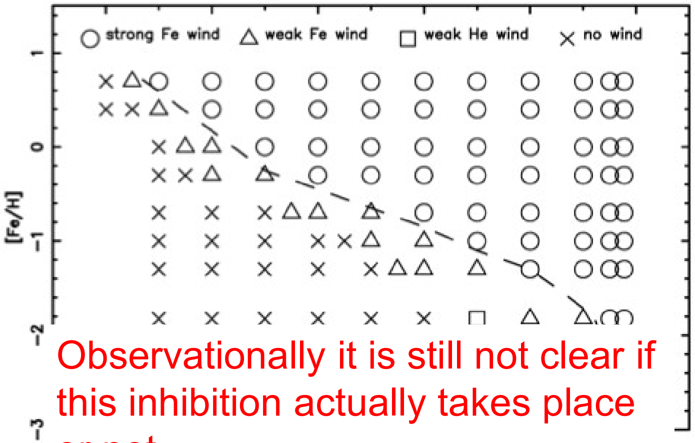
FIG. 4.—Late evolutionary path to an SN Ia explosion in our wind model

# Another way: RG+WD system
# (Symbiotic Star Chanel)

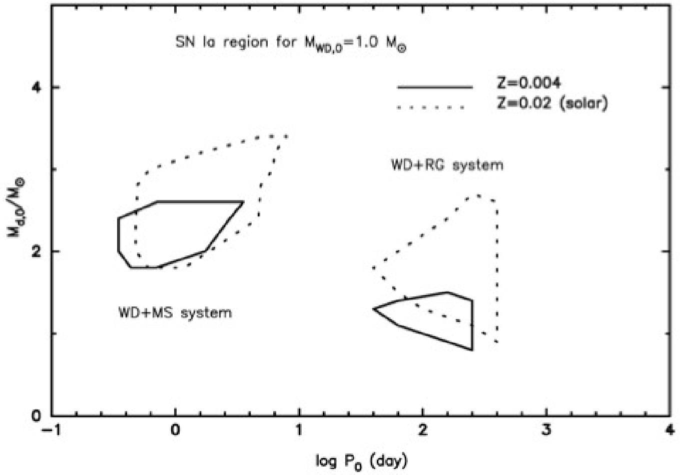


Symbiotic star➡

## Low metallicity inhibition of SNe Ia for the wind model (Kobayashi et al. 1998)

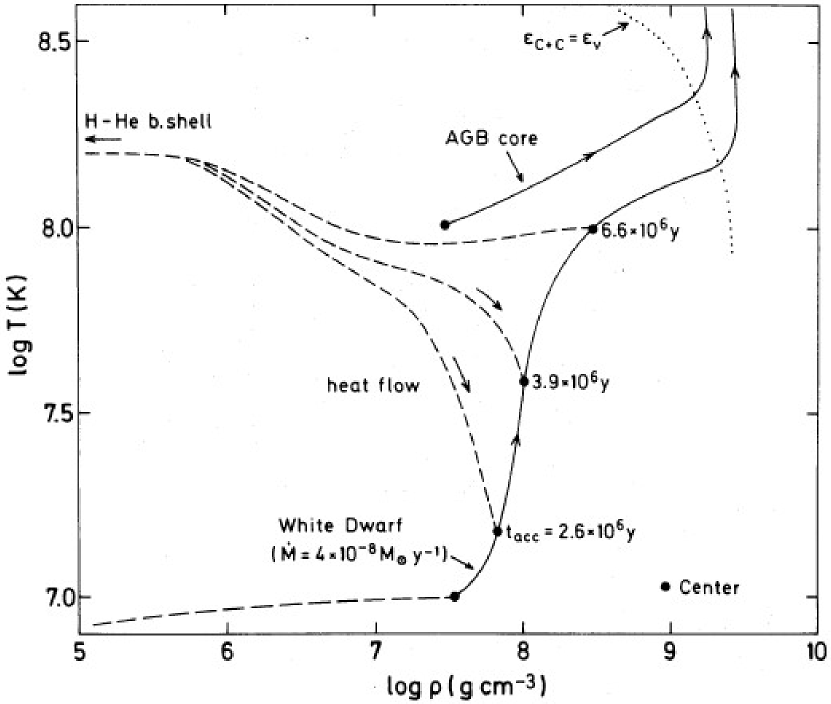


Observationally it is still not clear if this inhibition actually takes place or not.



## Type Ia SN

- C ignition at center
  - Cooling by neutrino emission
  - Degenerate electron :
    Runaway nuclear burning ➡explosion



# Neutrino emission

- When stellar temperature exceeds $10^9$ K, various neutrino emission processes occur.
  - Pair annihilation neutrinos $e^- + e^+ \rightarrow \nu + \overline{\nu}$

$$\varepsilon_\nu^{(\mathrm{pair})} = \begin{cases} \frac{4.9\times10^8}{\varrho} T_9^3 \mathrm{e}^{-11.86T_9} \quad , & \mathrm{T}_9 < 1 \\ \frac{4.45\times10^{15}}{\varrho} T_9^9 \quad , & \mathrm{T}_9 > 3 \end{cases}$$

  - Photo neutrinos $\gamma + e^- \rightarrow e^- + \nu + \overline{\nu}$

$$\varepsilon_\nu^{(\mathrm{phot})} = \varepsilon_1 + \varepsilon_2(\mu_\mathrm{e} + \bar{\varrho})^{-1} \ ,$$
$$\varepsilon_1 = 1.103 \times 10^{13} \varrho^{-1} T_9^9 \mathrm{e}^{-5.93/T_9} \ ,$$
$$\varepsilon_2 = 0.976 \times 10^8 T_9^8 (1 + 4.2T_9)^{-1} \ ,$$
$$\bar{\varrho} = 6.446 \times 10^{-6} \varrho T_9^{-1} (1 + 4.2T_9)^{-1}$$

  - Plasma neutrinos $\gamma_{plasmon} \rightarrow \nu + \overline{\nu}$

$$\omega_0^2 \frac{m_\mathrm{e}}{4\pi e^2 n_\mathrm{e}} = \begin{cases} 1 \ , & \text{non-degenerate} \\ \left[1 + \left(\frac{\hbar}{m_\mathrm{e}c}\right)^2 (3\pi^2 n_\mathrm{e})^{2/3}\right]^{-1/2} \ , & \text{degenerate} \end{cases} .$$

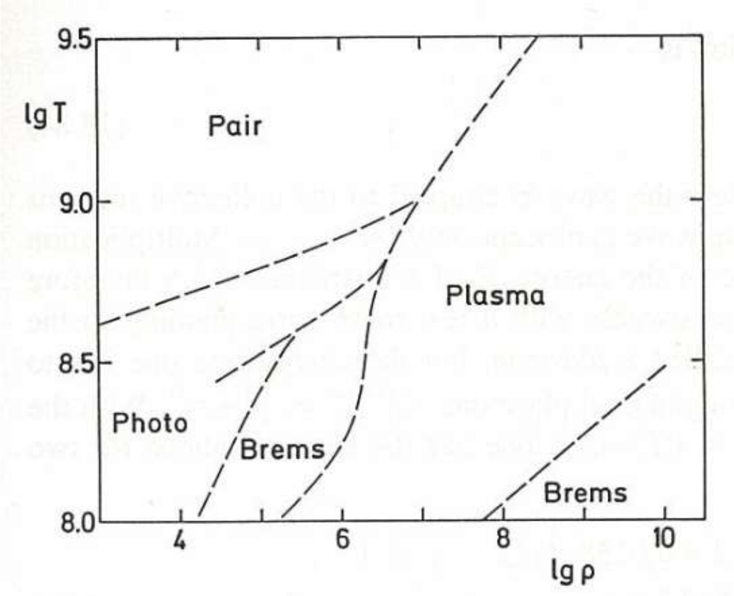
$$\gamma = \hbar\omega_0 / kT$$
$$\lambda = kT / m_e c^2$$

$$\varepsilon_\nu^{(\mathrm{plasm})} = 3.356 \times 10^{19} \varrho^{-1} \lambda^6 (1 + 0.0158\gamma^2) T_9^3 \quad , \gamma \ll 1$$
$$\varepsilon_\nu^{(\mathrm{plasm})} = 5.252 \times 10^{20} \varrho^{-1} \lambda^{7.5} T_9^{1.5} \mathrm{e}^{-\gamma} \quad , \gamma \gg 1 \quad ,$$

  - Bremsstrahlung: deceleration of an electron

$$\varepsilon_\nu^{(\mathrm{brems})} \approx 0.76 \frac{Z^2}{A} T_8^6$$

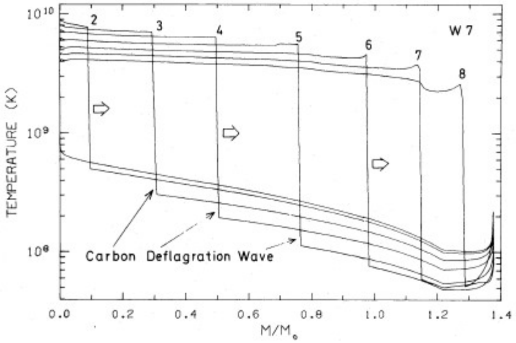
# Neutrino emission



# Ia型超新星

- 野本のW7モデル（炭素爆燃波モデル）
  c.f. 爆轟波（超音速）

密度が高い($> 10^9$ g cm$^{-3}$ )と核燃焼によるエネルギーがFermiエネルギーの20%程度にしかならず、強い衝撃波が発生しない。燃焼波面が対流によって伝播　（対流のMixing length が0.7 --- W7)



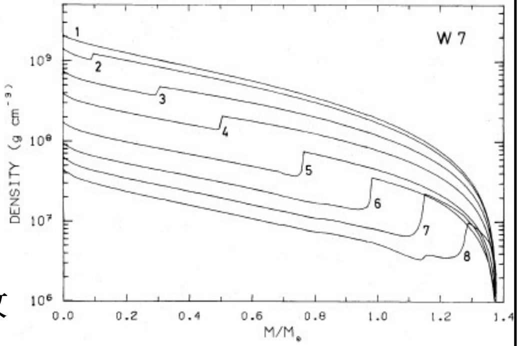
核燃焼による爆発エネルギーは
約$10^{51}$ erg

約$0.6M_\odot$ の$^{56}$Ni
が生成される

Ia型超新星の光源

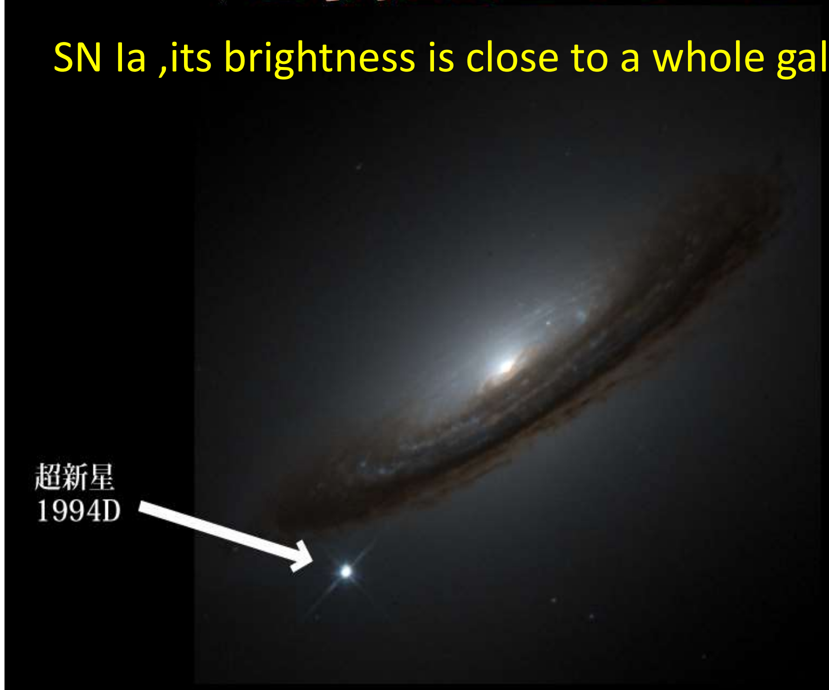
$^{56}$Niの崩壊エネルギー

$^{56}$Ni　→ $^{56}$Co　→ $^{56}$Fe
$0.6M_\odot$ の$^{56}$Niは観測される明るさと一致



# 超新星 1994D

SN Ia ,its brightness is close to a whole galaxy



# Recent status of SN Ia model

(becoming more confusing lately)

- Long arguments about progenitor system
  - Merging of two WDs :
    Double degenerate (DD) model
  - Binary accretion (WD wind) model : Single degenerate (SD) model
  - Next page for detail.
- Also arguments about explosion model
  - ~~Detonation model~~ (observationally ruled out)
  - Deflagration model (slower than sound velocity)
  - Delayed Detonation model
    (Delayed Detonation – DDT model)
  - DDT model fits observations better (partly because the model includes more parameters). However, current simulations can't confirm the transition to Detonation.
  - Very recently another explosion model, called the violent merger model, was proposed (see below). In this model pure detonation explains SNe Ia.

# SD or DD

- Good for SD (or not good for DD)
  - There are some binary systems that seem to have a WD close to $M_{ch}$ and getting mass accretion from their companion.
  - There are a few SNe Ia that have Hydrogen feature. This may be an evidence for mass accretion before explosion. (However, most SNe Ia doesn't have any Hydrogen feature, so this may be against the SD model.
- Not good for SD (or not bad for DD)
  - Observed delay time (between the star formation and SN Ia explosion) distribution seem to be more consistent with the DD model: There appear to exist many SN with short delay time.
    - (but in SD, newer path has been proposed to explain the short delay time.
  - There appear to be no evidence for low metallicity inhibition, that's a prediction of the SD model.
    - More observations are required for the concrete conclusion.
  - So far, no evidence of the existence of a companion star, in the spectra of SNe and images in the supernova remnants.
    - Need more observations

# SD or DD

- Super–Chandra SNe Ia ?
  - Very bright, and with a reasonable estimate the total mass of the ejecta should exceed the Chandrasekharl limit.
  - Naively it means the evidence of DD because the sum of two WDs can exceed the limit.
  - However, it is not so simple (next paragraph
  - SD model can explain super Chandra progenitor if the progenitor WD is supported with vary rapid core rotation (though there are no observational evidence that such a massive WD can really exist.
- Theoretical aspects:
  - It was once considered that DD explosion doesn't occur, because after the merging one of a WD is broken and forms a massive accretion disk around the other WD. Then with the rapid mass accretion, C–burning ignites at the surface of the WD and then explosion is too weak to disrupts the star. Such a system is considered to form a neutron star without SNe Ia explosion.
  - However, recently another scenario was proposed for the DD model: Violent merger model.

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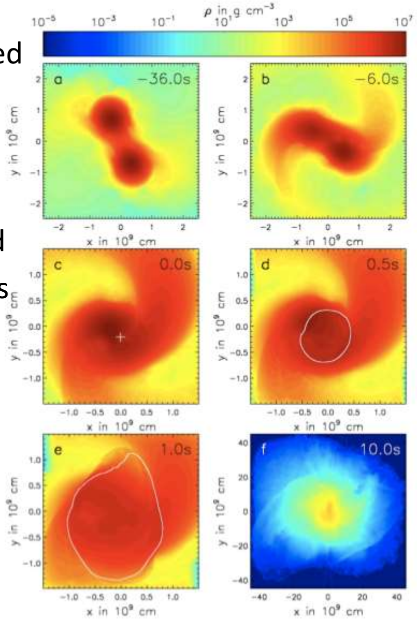
# Violent merger model of DD

Pakmor et al. 2010, 2012, 2013

- 2010: Sub-luminous type Ia supernovae from the mergers of equal-mass white dwarfs with mass 0.9 Msun
  - Nearly equal mass WDs merge violently
  - Hot spots appear in a high density region, and carbon detonation ignites here.

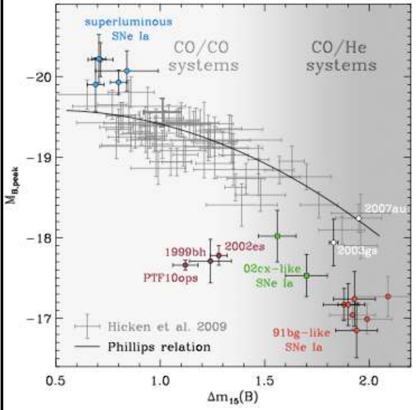


- Previously it was considered that detonation is not a burning mechanism for SNe Ia, because if it occurs at center, it burns entire star and inconsistent with observations showing some unburned materials.

## Violent merger model of DD
Pakmor et al. 2010, 2012, 2013

- 2012: Normal Type Ia supernovae from violent mergers of white dwarf binaries of mass 0.9 Msun and 1.1 Msun.
- 2013: Helium-ignited violent mergers as a unified model for normal and rapidly declining Type Ia Supernovae
- using a moving-mesh code that allows for the inclusion of thin helium (He) shells (0.01 M ☉) on top of the WDs, at an unprecedented numerical resolution. The accretion of He onto the primary WD leads to the formation of a detonation in its He shell. This detonation propagates around the CO WD and sends a converging shock wave into its core, known to robustly trigger a second detonation, as in the well- known double-detonation scenario for He-accreting CO WDs. the required He-shell mass is significantly smaller, and hence its burning products are unlikely to affect the optical display of the explosion. We show that this scenario, which works for CO primary WDs with CO- as well as He-WD companions, has the potential to explain the different brightness distributions, delay times and relative rates of normal and fast declining SNe Ia.

## Pakmor et al. 2013



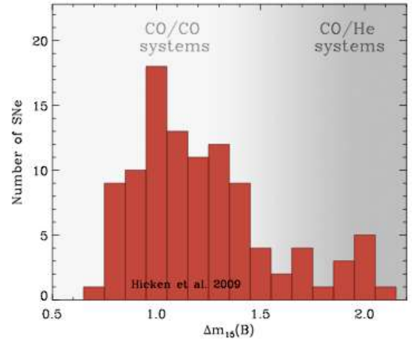


**Figure 5.** Histogram of the observed number of SNe Ia as a function of $\Delta m_{15}(B)$ (data from table 9 of Hicken et al. (2009)). The distribution shows some indication for bimodality which might be identified with the two different companion populations in our model (illustrated by the background-color gradient as in Fig 4).

## Summary

- Violent merger can explain all observed SNe Ia? -- not sure yet

- Can explain observed SN rate ?

- So far, VM can't explain super-chandra type. (DD rotation model can?

- Some are DD some are SD ??

- Uncertainty in numerical resolutions and assumptions.

## D6 model

the dynamically driven double-degenerate double-detonation model

（近年流行っている
Sub-Chandrasekhar Mass model）

例：　Tanikawa + 2019, ApJ 885, 103

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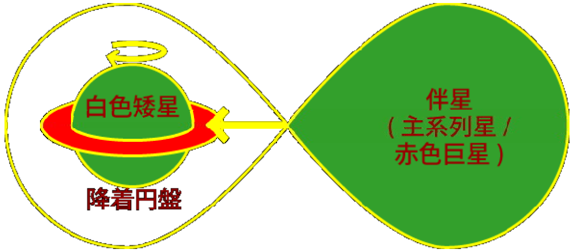
## Ia型超新星の理論の状況

- 母天体モデルに関する長い論争、
  - 2つの白色矮星の合体モデル
  - 連星、降着モデル

- 爆発モデルに関する論争
  - 爆燃波モデル（音速以下）
  - 遅延爆轟波モデル（途中から速以上）
    （Delayed Detonation – DDT モデル）
  - DDTのほうが観測によく合うが、問題はシミュレーションでDetonationへの遷移が簡単に起きない
  - いくつかスーパーチャンドラの示唆
    - どのように作るのか？



Ia型超新星の母天体モデル

## Ia型超新星

宇宙の標準光源： 光度曲線から絶対的な明るさ

↓

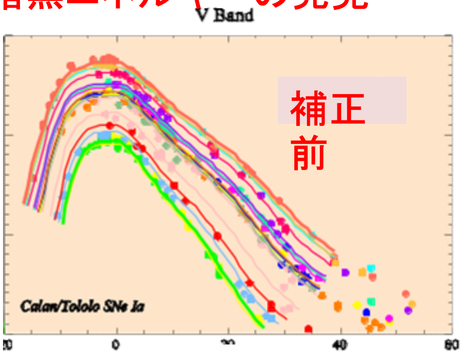
たいへん遠方まで明るさ（距離）が測れる

↓

宇宙膨張の速さの変化が測れる

＝宇宙に何が詰まっているのかわかる

暗黒エネルギーの発見

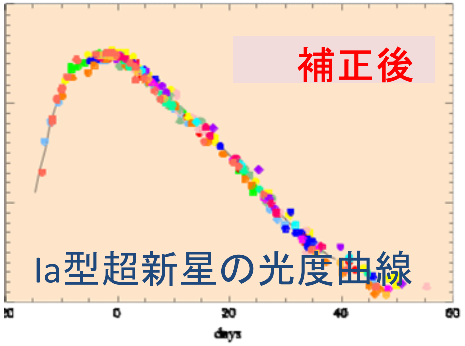


絶対等級

時間

–幅と絶対等級が対応（経験則）

Phillips relation という



絶対等級

Ia型超新星の光度曲線

–この補正の起源の理論的裏付けが無い

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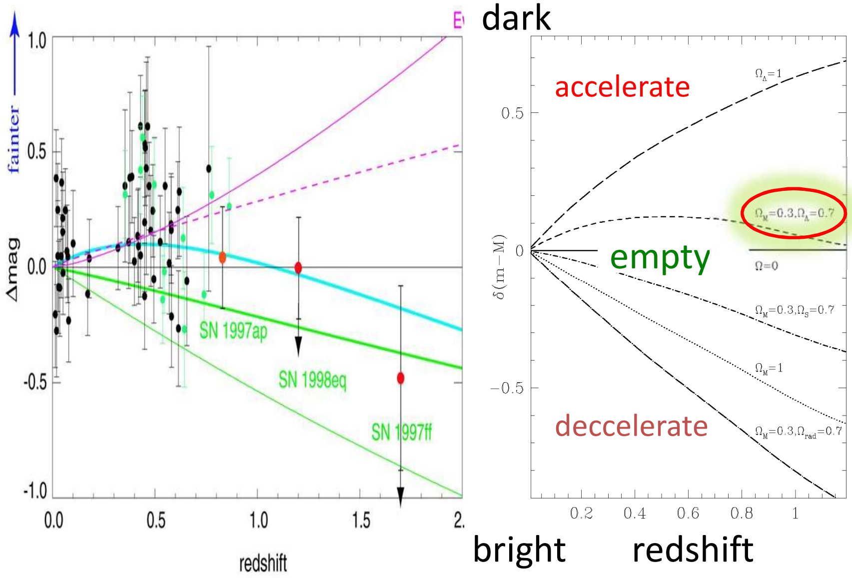
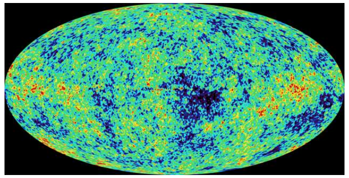
## Our universe is accelerating！

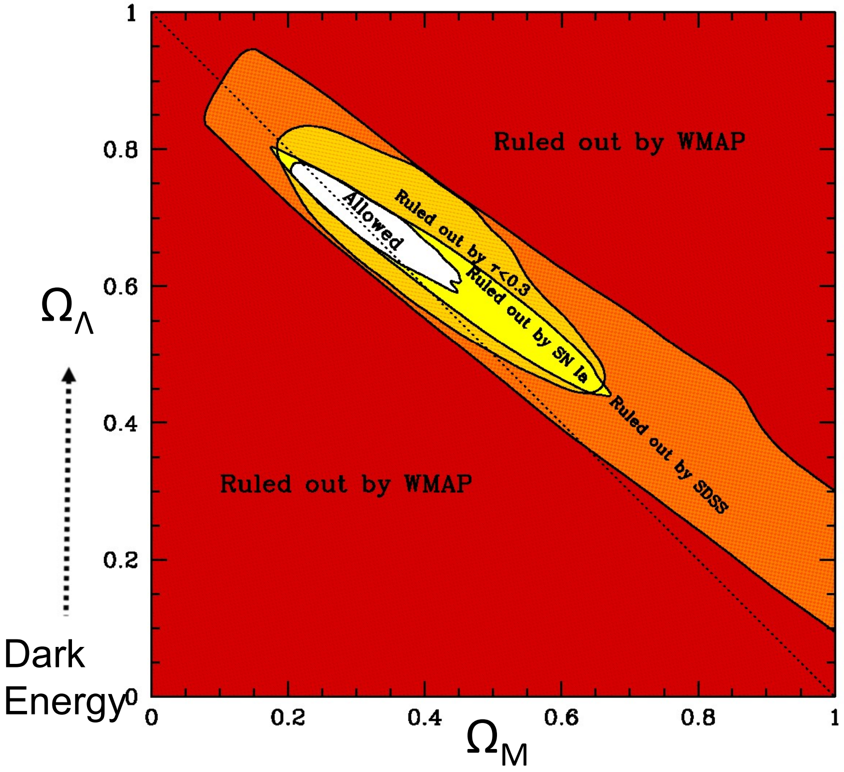
Perlmutter et al.1999, Riess et al., Schmidt et al., 1998



# With other observations

CMB observation Max Tegmark et al.2003





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# 宇宙の組成
# 現在の標準宇宙モデル

- ➲ 元素（バリオン）は宇宙の構成要素の約5%にすぎない
- ➲ 残りは，未知の暗黒物質および暗黒エネルギー