EOS and Stability of Stars

$$E = \int u\, dm - \int \frac{Gm}{r} dm = <u> M - c_1 \frac{GM^2}{R}$$

u:内部エネルギー

(変分法によりMとρ_cの関係を求めたい)

$P = K\rho^{\gamma}$のEOSを考えると

$$dq = du + Pdv = du - \frac{P}{\rho^2} d\rho = 0 \qquad (断熱)$$

$$\Rightarrow du = K\rho^{\gamma-2} d\rho \Rightarrow u = \frac{K\rho^{\gamma-1}}{\gamma-1}$$

$\Rightarrow <u> = c_2 K \rho_c^{\gamma-1}$, また$R = c_3 (M/\rho_c)^{1/3}$

(ポリトロープ)

$$\therefore E = c_2 K \rho_c^{\gamma-1} M - c_4 G M^{5/3} \rho_c^{1/3}$$

平衡解は$\frac{dE}{d\rho_c} = 0 \Rightarrow M \propto \rho_c^{\frac{3}{2}(\gamma - \frac{4}{3})}$

$$\therefore \frac{dM}{d\rho_c} > 0 \ (stable) \Rightarrow \gamma > \frac{4}{3}$$

$$< 0 \ (unstable) \Rightarrow \gamma < \frac{4}{3}$$

Feの光分解

$$\gamma + {}^{56}_{26}\mathrm{Fe} \Leftrightarrow 13\,{}^{4}\mathrm{He} + 4\mathrm{n}$$

(Q - value : energy required for this process)

$$\mathrm{Q} = \mathrm{c}^2 (13 m_\alpha + 4 m_n - m_{Fe}) = 124.4\,\mathrm{Mev}$$

$\mu_{\mathrm{Fe}} = 13\mu_\alpha + 4\mu_n$ (chemical potential)

非相対論的 Maxwell - Boltzman gas

$$n = g \left(\frac{mkT}{2\pi \bar{h}^2} \right)^{3/2} \exp\left(\frac{\mu - mc^2}{kT} \right) \qquad (\bar{h}^2 \equiv \hbar^2)$$

$$\frac{\mu_i - m_i c^2}{kT} = \ln \left[\frac{\mathrm{n}_i}{g_i} \left(\frac{2\pi \bar{h}^2}{m_i kT} \right)^{3/2} \right]$$

$$g_i = \sum_r (2 I_r + 1) \quad e^{-E_r/kT}$$

(I_r : spin of the r - th excited state)

$$g_\alpha = 1 \ (I = 0), \ g_n = 1 \ (I = 1/2), \ g_{Fe} \cong 1.4$$

$$\therefore \frac{n_\alpha^{13} n_n^{4}}{n_{Fe}} = \frac{g_\alpha^{13} g_n^{4}}{g_{Fe}} \left(\frac{kT}{2\pi \bar{h}^2} \right)^{24} \left(\frac{m_\alpha^{13} m_n^{4}}{m_{Fe}} \right)^{3/2} e^{-Q/kT}$$

Feの光分解

- 分解前にこの物質がほぼ^{56}Feから成っているとすると、
  - $n_n = 4n_\alpha/13$
  - ^{56}Feの半分が分解するとき、
    - Log ρ= 11.62+1.5 logT_9-39.17/T_9
- 更なる分解反応　$\gamma + {}^4\mathrm{He} \Leftrightarrow 2\mathrm{p} + 2\mathrm{n}$ のQ-value はQ' =28.30 MeV で^{4}Heまでの分解よりも起こり易そうに見える、が、

Required energy per new particle

ΔN = 4-1=3, Q'/ ΔN=9.5 Mev は、

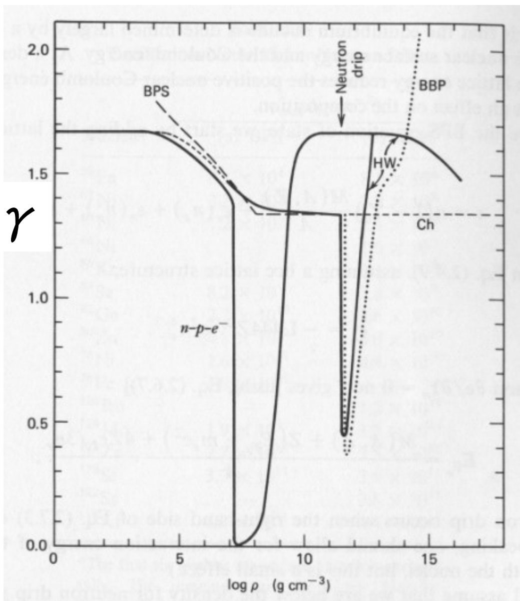
^{4}Heまでの分解の時の

Q/ΔN=Q/(13+4-1)=7.7MeV より

大きいため、^{4}Heの分解はより高温で

起きる。

EOS and γ



こういう図の新しい例は数ページ後に

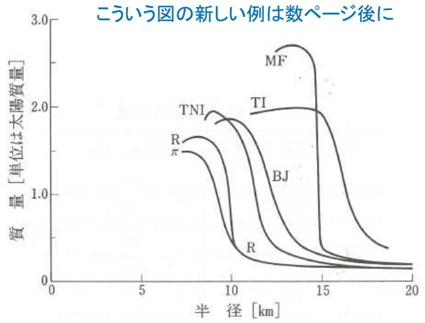


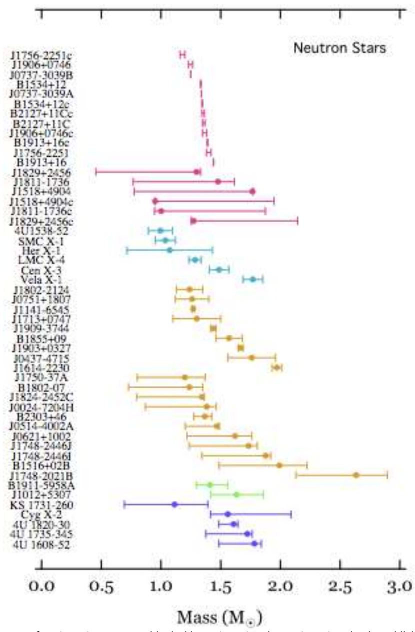
図 4 中性子星の質量と半径。各曲線についている英文字やギリシャ文字は用いられた状態方程式の略名である。[G. Baym and C. Pethick: Annual Review of Astronomy and Astrophysics **17** (1979) 415]

TOV(Tolman - Oppenheimer - Volkoff)方程式

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

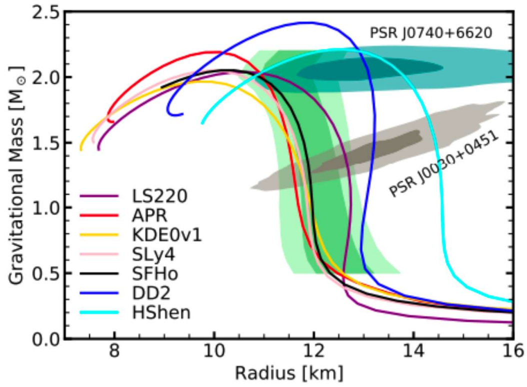
$$\frac{dP}{dr} = -\frac{\rho m}{r^2}\left(1+\frac{P}{\rho}\right)\left(1+\frac{4\pi P r^3}{m}\right)\left(1-\frac{2m}{r}\right)^{-1}$$

Ozel et al. 2012



The masses of neutron stars measured in double neutron stars (magenta; categories Ia and IIa), in eclipsing binaries with primarily high mass companions (cyan; category IV; these are the numerical values from Rawls et al. 2011 given in column 2 of Table 6), with white dwarf companions (gold; categories Ib and IIb), with optical observations of the white dwarf companions (green; category III), and in accreting bursters (purple; category V).

中性子星用の新しい状態方程式(EOS)と観測範囲



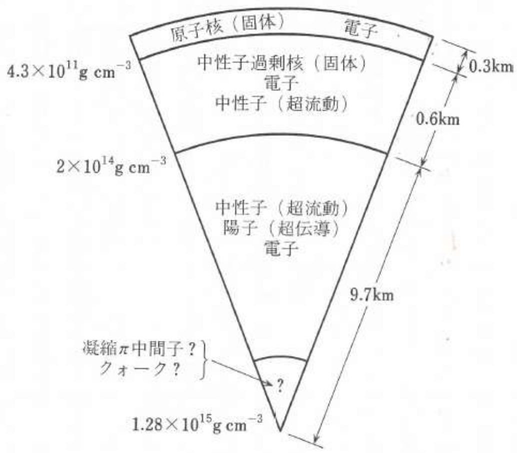


図 7 中性子星の断面図