Polytrope & White dwarfs

$K = c_1 A^{-2} G\rho_c^{\frac{n-1}{n}}$ (KH 19.9),

c_i : non dimensional constants

$\& \ \rho_c \propto \bar{\rho} \propto MR^{-3} = M\left(\frac{A}{z_n}\right)^3$

$\to A = c_2 \rho_c^{1/3} M^{-1/3}$

$\to K = c_3 M^{2/3} G\rho_c^{1-\frac{1}{n}-\frac{2}{3}} = c_3 M^{2/3} G\rho_c^{\frac{1}{3}-\frac{1}{n}}$,

also : $P_c = K\rho_c^{1+\frac{1}{n}} = c_3 G M^{2/3} G\rho_c^{\frac{4}{3}}$

$P_c = 0.48\ GM^{2/3} G\rho_c^{\frac{4}{3}}$ $\quad (n = \frac{3}{2})$ (1.21)

$= 0.36\ GM^{2/3} G\rho_c^{\frac{4}{3}}$ $\quad (n = 3)$ (1.22)

Lane – Emden equation

$$\frac{1}{z^2}\frac{d}{dz}\left(z^2\frac{dw}{dz}\right) + w^n = 0$$

$$M = 4\pi\rho_c R^3\left(-\frac{1}{z}\frac{dw}{dz}\right)_{z=z_n}$$

$$\frac{\bar{\rho}}{\rho_c} = \left(-\frac{3}{z}\frac{dw}{dz}\right)_{z=z_n}$$

n	z_n	$\left(-z^2\frac{dw}{dz}\right)_{z=z_n}$	$\varrho_c/\bar{\varrho}$
0	2.4494	4.8988	1.0000
1	3.14159	3.14159	3.28987
1.5	3.65375	2.71406	5.99071
2	4.35287	2.41105	11.40254
3	6.89685	2.01824	54.1825
4	14.97155	1.79723	622.408
4.5	31.8365	1.73780	6189.47
5	∞	1.73205	∞

準静的収縮
Quasi-static Contraction

$E_{GR} = -\int_0^R \frac{Gm\rho}{r}4\pi r^2 dr = -\frac{3GM^2}{5R}$ $\quad$ ($\rho =$ 一定の時)

一般に $\quad E_{GR} \approx -\frac{GM^2}{R}$

(1.5) $\to \quad P_c \propto \langle P \rangle \approx \frac{GM^2}{4\pi R^4}$ $\quad$ (1.8)

ρ_cは星の平均密度に比例 $\quad \rho_c \propto \frac{M}{\frac{4}{3}\pi R^3}$

$\frac{P_c}{\rho_c} \propto \frac{GM}{R}$ $\quad$ (1.9), $\quad \frac{P_c^3}{\rho_c^4} \propto G^3 M^2$ $\quad$ (1.10)

理想気体状態方程式 $\quad P_c = \frac{k}{\mu m_u}\rho_c T_c$

$+$(1.10) $\to \frac{T_c^3}{\rho_c} \propto G^3 M^2$ $\quad$ (1.11)

(1.11)式 $\Rightarrow M$一定のとき$\rho_c \propto T_c^3$

Quasi-static contraction

エントロピー(単位体積あたりの)

$s = \frac{k}{\mu m_u}\ln(T^{3/2}/\rho) + C_1$,

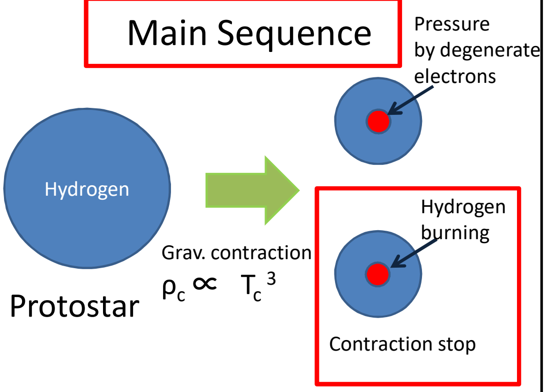
$+$(1.11) $\to$

$(\frac{\mu m_u}{k})s_c = \ln(\frac{M^2}{T_c^{3/2}}) + C_2 = \ln(\frac{M}{\rho_c^{1/2}}) + C_3$ $\quad$ (1.12)

(1.12)式 $\Rightarrow M$一定でs_cが減少するとT_c上昇

比熱は負 $\quad C_g = \frac{ds_c}{d\ln T_c} = -1.5\frac{k}{\mu m_\mu}$ $\quad$ (1.13)

Main Sequence



- **Maximum Stellar temperature determined by the degenerate pressure (rough estimate)**

(1.6)ビリアル定理 $2E_{KE} + E_{GR} = 0$

$$E_{GR} \approx -\frac{GM^2}{R}, \quad E_{KE} = \frac{3}{2}NkT \rightarrow kT \approx \frac{GMm_u}{3R} \approx Gm_u M^{2/3}\rho^{1/3}$$

M/N

＋縮退圧の効く密度：ド ・ブロイ波長

$$\lambda \approx \frac{h}{\sqrt{m_e kT}}, \quad \rho \approx \frac{m_u}{\lambda^3} \approx m_u \frac{(m_e kT)^{3/2}}{h^3}$$

$M/R^3 \sim \rho$

$$kT_{max} \approx \left[\frac{G^2 m_u^{8/3} m_e}{h^2}\right] M^{4/3} \qquad (1.14)$$

Hydrogen Burning (KH193)

- (1.14)$\rightarrow$ $M < 0.08M_\odot - T_{max} <\sim 10^7$ K
  Hydrogen can't be ignited.
  ➡ Brown dwarfs (褐色矮星, supported by the degenerate pressure)
- $T \sim 1.5\mathrm{x}10^7$ K: **pp (proton-proton) chain**

$p + p \rightarrow d + e^+ + \nu_e$

$p + d \rightarrow {}^3He + \gamma$

1) ${}^3He + {}^3He \rightarrow {}^4He + p + p$ (pp1)

2) ${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

2a) $e^- + {}^7Be \rightarrow {}^7Li + \nu_e$

$p + {}^7Li \rightarrow {}^4He + {}^4He$ (pp2)

2b) $p + {}^7Be \rightarrow {}^8B + \gamma$

${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$

${}^8Be^* \rightarrow {}^4He + {}^4He$ (pp3)

$\varepsilon_{pp} \sim 2.36\mathrm{x}10^6 X_H^2 \rho T_6^{-2/3}$
$\mathrm{x}\exp(-33.8\, T_6^{-1/3})$ erg g^{-1}sec^{-1}

Solar neutrino (KH338)

- **pp (proton-proton) chain**

$p + p \rightarrow d + e^+ + \nu_e$

$p + d \rightarrow {}^3He + \gamma$

1) ${}^3He + {}^3He \rightarrow {}^4He + p + p$ (pp1)

2) ${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

2a) $e^- + {}^7Be \rightarrow {}^7Li + \nu_e$

$p + {}^7Li \rightarrow {}^4He + {}^4He$ (pp2)

2b) $p + {}^7Be \rightarrow {}^8B + \gamma$

${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$

${}^8Be^* \rightarrow {}^4He + {}^4He$ (pp3)

Process	Neutrino flux (10^{14} m^{-2} s^{-1})	Maximum neutrino energy (MeV)
$p + p \rightarrow d + e^+ + \nu_e$	6.0 (1 ± 0.02)	0.420
$e^- + {}^7Be \rightarrow {}^7Li + \nu_e$	0.47 (1 ± 0.15)	0.861
${}^8B \rightarrow {}^8Be + e^+ + \nu_e$	5.8 × 10^{-4} (1 ± 0.37)	15
${}^{13}N \rightarrow {}^{13}C + e^+ + \nu_e$	0.06 (1 ± 0.50)	1.199
${}^{15}O \rightarrow {}^{15}N + e^+ + \nu_e$	0.05 (1 ± 0.58)	1.732

$\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e^+$ (detectable only above 0.81MeV)
Predicted 7.9±2.6 SNU, Observed 2.5±0.4 SNU

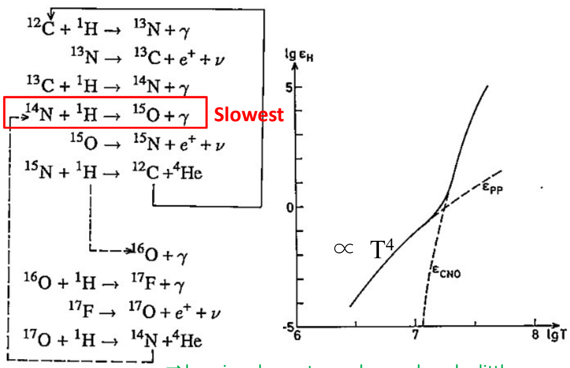
Kamioka $\nu_e + {}^2H \rightarrow p + p + e^-$, $\nu_x + {}^2H \rightarrow n + p + \nu_x$
Result ν_e changes to ν_{mu} & ν_{tau}

Discovery of the neutrino oscillation (& neutrino mass)

Hydrogen burning

- $M > 1\, M_\odot$, at $T \sim 2\mathrm{x}10^7$ K

CNO cycle (KH196)



⇒heavier elements can be produced a little

$\varepsilon_{CNO} \sim 8.67\mathrm{x}10^{27} X_{CNO} X_H \rho T_6^{-2/3}$
$\mathrm{x}\exp(-152.3\, T_6^{-1/3})$ erg g^{-1}sec^{-1}
$\propto \rho T^{16}$ at $T \sim 2\mathrm{x}10^7$ K

${}^{12}C : {}^{13}C : {}^{14}N : {}^{15}N = 4 : 1 : 95 : 0.004$