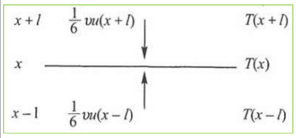
Energy transfer in a star

- **Radiation (rondom motion of photons)**
  - u(*x*) : thermal energy per unit volume at *x*
  - Assuming that 1/6 of the photons moves toward *x* direction, and letting the mean free path *l*

Heat flux density: $j(x) =$



$$\frac{1}{6}vu(x-l)-\frac{1}{6}vu(x+l)$$

$$=-\frac{1}{3}vl\frac{du}{dx}=-\frac{1}{3}vl\frac{du}{dT}\frac{dT}{dx}$$

$$=-K\frac{dT}{dx} \quad (K=\frac{1}{3}vlC)$$

Heat conduction coefficient

Heat capacity per unit volume

Radiative diffusion:

$$u_r = aT^4, C_r = 4aT^3, K_r \approx \frac{4}{3}claT^3$$

$$l=(\rho\kappa)^{-1} \quad (\kappa : \text{opacity})$$

$$\rightarrow j(x) = -\frac{4ac}{3}\frac{T^3}{\rho\kappa}\frac{dT}{dx}$$

Energy transfer in a star

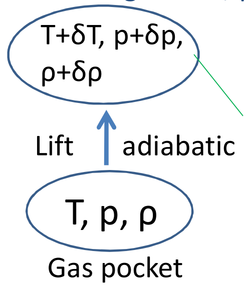
- When radiative diffusion dominates, energy flow rate *L(r)* is

$$L(r) = 4\pi r^2 j(r) = -4\pi r^2 \frac{4ac}{3}\frac{T^3}{\rho\kappa}\frac{dT}{dr}$$

$$\rightarrow \left[\frac{dT}{dr}\right]_{rad} = -\frac{3\rho\kappa}{4acT^3}\frac{L}{4\pi r^2} \quad (1.15)$$

- Transfer by convection

Surroundings: T+ΔT, p+ΔT, ρ+ΔT



Keep moving up, If the lifted gas pocket has lower density than the surroundings.

Condition for convection
δρ < Δρ (|δρ| > |Δρ|)

Energy transfer inside a star

- Transfer by convection

$$\text{adiabatic} \rightarrow p \propto \rho^\gamma \rightarrow \frac{\delta\rho}{\rho} = \frac{1}{\gamma}\frac{\delta p}{p}$$

$$\text{ideal gas } \rho \propto \frac{p}{T} \rightarrow \frac{\Delta\rho}{\rho} = \frac{\Delta p}{p} - \frac{\Delta T}{T}$$

Convective condition
δρ < Δρ (|δρ| > |Δρ|)

Pressure inside the gas pocket quickly is balanced with outside: δp = Δp

$$\rightarrow \frac{1}{\gamma}\frac{\delta p}{p} < \frac{\Delta p}{p} - \frac{\Delta T}{T} \rightarrow \frac{\Delta T}{T} < \frac{\gamma-1}{\gamma}\frac{\Delta p}{p}$$

$$\rightarrow \frac{dT}{dr} < \frac{\gamma-1}{\gamma}\frac{T}{p}\frac{dp}{dr} \equiv \left[\frac{dT}{dr}\right]_{crit} \quad (\text{Note}: \frac{dT}{dr} < 0)$$

Summary of Energy transfer equations

- Transfer efficiency by convection is very high, so in the convective region: $\frac{dT}{dr} = \left[\frac{dT}{dr}\right]_{crit}$

$$\text{*if} \left|\frac{dT}{dr}\right| < \left|\frac{dT}{dr}\right|_{crit} \quad (radiative)$$

$$\frac{dT}{dr} = -\frac{3\rho\kappa}{4acT^3}\frac{L}{4\pi r^2} \quad (1.15)$$

$$\text{*if} \left|\frac{dT}{dr}\right| > \left|\frac{dT}{dr}\right|_{crit} \quad (convective)$$

$$\frac{dT}{dr} = \frac{\gamma-1}{\gamma}\frac{T}{p}\frac{dp}{dr}\left(\equiv \left[\frac{dT}{dr}\right]_{crit}\right)$$

$$= -\frac{\gamma-1}{\gamma}\frac{T}{p}\frac{Gm\rho}{r^2} \quad (1.16)$$

$$(\text{equation for hydrostatic } \frac{dp}{dr} = -\frac{Gm\rho}{r^2})$$

Energy transfer equation (supplement)

$$\left(\frac{dT}{dr}\right)_{crit} = \frac{\gamma-1}{\gamma}\frac{T}{P}\frac{dP}{dr}$$

$$\rightarrow \frac{P}{T}\left(\frac{dT}{dP}\right)_{crit} = \left(\frac{d\ln T}{d\ln P}\right)_{crit} = \frac{\gamma-1}{\gamma}$$

$$\text{define } \frac{d\ln T}{d\ln P} \equiv \nabla, \text{ then } \nabla_{crit} = \frac{\gamma-1}{\gamma}$$

$$\text{(1.15 when radiative)} \rightarrow \nabla_{rad} = \frac{P}{T}\frac{dr}{dP}\frac{dT}{dr} = \frac{3\kappa LP}{16\pi acT^4 Gm}$$

therefore, energy transfer eqation is

when radaiative ($\nabla < \nabla_{crit}$) : $\nabla = \nabla_{rad}$

when convective : $\nabla = \nabla_{crit}$

* About Le doux criterion

エネルギー保存の式

- 完全に静的(stationary)な場合, エネルギー生成率εを持つ厚さdrの球殻を通過するときのenergy flow rate (luminosity) の変化量は

dL = 4πr²ρ ε dr = ε dm 又は

- 一般に

$$dq = \left(\varepsilon - \frac{\partial L}{\partial m}\right)dt = Tds \rightarrow T\frac{\partial s}{\partial t} = \varepsilon - \frac{\partial L}{\partial m}$$

$\varepsilon = \varepsilon_n - \varepsilon_\nu, \quad \varepsilon_g \equiv -T\dfrac{\partial s}{\partial t}$ とすると

ε_n : nuclear energy generation rate

ε_ν : neutrino energy loss rate

$$\frac{\partial L}{\partial m} = \varepsilon_n - \varepsilon_\nu + \varepsilon_g$$

(1.17 : エネルギー保存の式)

Energy conservation

Modifying ε_g into a more convenient form (using the first law of thermodynamics)

$$Tds = du + Pdv = \left(\frac{\partial u}{\partial T}\right)_v dT + \left[P + \left(\frac{\partial u}{\partial v}\right)_T\right]dv$$

$= c_v dT - \dfrac{P\delta}{\rho^2\alpha}d\rho \quad$ ($v = 1/\rho$: specific volume, c_v : specific heat at constant volume)

$$\alpha \equiv \left(\frac{\partial \ln\rho}{\partial \ln P}\right)_T, \quad \delta \equiv -\left(\frac{\partial \ln\rho}{\partial \ln T}\right)_P$$

$$\varepsilon_g = -T\left(\frac{\partial s}{\partial t}\right) = -c_v\frac{\partial T}{\partial t} + \frac{P\delta}{\rho^2\alpha}\frac{\partial\rho}{\partial t}$$

$$= -c_p T\left(\frac{1}{T}\frac{\partial T}{\partial t} - \frac{\nabla_{ad}}{P}\frac{\partial P}{\partial t}\right) \quad (1.18)$$

c_p : specific heat at constant pressure

$$\nabla_{ad} \equiv \left(\frac{\partial \ln T}{\partial \ln P}\right)_s = \frac{P\delta}{T\rho c_p}$$

: adiabatic temperature gradient (1.19)

星の進化計算
Full set of equations

- Rewrite equations as a function of *m*

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2\varrho},$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2}, \quad \text{(with acceleration term)}$$

$$\frac{\partial L}{\partial m} = \varepsilon_n - \varepsilon_\nu - c_P\frac{\partial T}{\partial t} + \frac{\delta}{\varrho}\frac{\partial P}{\partial t}$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla, \quad \left(\nabla = \nabla_{rad} = \frac{3\kappa LP}{16\pi acT^4 Gm}, \nabla = \nabla_{convec}\right)$$

$$\frac{\partial X_i}{\partial t} = \frac{m_i}{\varrho}\left(\sum_j r_{ji} - \sum_k r_{ik}\right)$$

(境界条件 KH p.93~)
↓

- Usually solved by the Henyey method
  - Non-liner boundary condition problem（boundary conditions depend on time implicitly）
  - Define δ**y** as the difference between the temporal and true solution. Then linearize the equation for δ**y** to solve
  - Repeat (iterate) this procedure until δ**y** becomes sufficiently small