大質量星の特徴の一つ：圧力に輻射の影響が強い

→ Eddington のStandard model が有用

Eddington Luminosity

- Near the surface of a star, matter has low density and non-degenerate, then P = P_{gas} + P_{rad} = k ρ T / (μ m_u) + a T^4 / 3.
- We consider a region where the density is too low to transfer energy by convection. Then the energy transfer equation is written in the form,

$$\frac{dT}{dr} = -\frac{3\rho\kappa}{4acT^3}\frac{L_r}{4\pi r^2} \quad (1.12). \quad (L_r = \text{luminosity, } \kappa\text{=opacity})$$

Using the relation $P_{\rm rad} = \frac{aT^4}{3}$, $\frac{dP_{\rm rad}}{dr} = -\frac{\rho\kappa}{c}\frac{L_r}{4\pi r^2}$.

the hydrostatic equation leads

$$\frac{dP}{dr} = \frac{dP_{\rm gas}}{dr} + \frac{dP_{\rm rad}}{dr} = \frac{dP_{\rm gas}}{dr} - \frac{\rho\kappa}{c}\frac{L_r}{4\pi r^2} = -\frac{\rho G m}{r^2} \quad (1.13),$$

Since $\frac{dP_{\rm gas}}{dr} < 0$ always, (1.13) leads $L_r < \frac{4\pi c G m}{\kappa}$.

At the stellar surface this inequality leads ($L_{\rm Edd}$ is the Eddington luminosity),

$$L < \frac{4\pi c G M}{\kappa_s} \equiv L_{\rm Edd} \quad (1.14),$$

here L is the stellar luminosity at surface and κ_s is the opacity at surface which is in many cases the electron scattering opacity.

Eddington's standard model

- In early stages of evolution for M > ~ 1$M_\odot$ stars, stellar interior is non-degenerate and the pressure can be written as
  P = P_{gas} + P_{rad} = k ρ T / (μ m_u) + a T^4 / 3 (1.15),
  where k = 1.38 x 10^{-16} erg K^{-1} = Boltzmann constant, m_u = 1.66053 x 10^{-24} g = the atomic mass unit, a = 7.56464 x 10^{-15} erg cm^{-3} K^{-4} = the radiation density constant, and μ = the molecular weight.
- We define the ratio of gas pressure to the total pressure β as,
  β ≡ P_{gas} / P then 1-β ≡ P_{rad} / P (1.16).
- From (1.8) and (1.9) we have
  P = P_{gas} + P_{rad} = k ρ T / (μ m_u) + a T^4 / 3 = k ρ T / (μ m_u β) (1.17).
- Deleting 'T' from this equation using a relation
  1-β = P_{rad} / P = aT^4/(3P), we have

$$P = \left(\frac{3k^4}{a\mu^4 m_u^4}\right)^{1/3}\left(\frac{1-\beta}{\beta^4}\right)^{1/3}\rho^{4/3} \quad (1.18).$$

(KH219, 226にも一応書いてある)

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Eddington's standard model

- He solved the energy transfer equation by radiation,

as follows. $$\frac{dT}{dr} = -\frac{3\rho\kappa}{4acT^3}\frac{L_r}{4\pi r^2} \quad (1.19) \quad (L_r = \text{luminosity, } \kappa\text{=opacity})$$

From $P_{\rm rad} = \frac{aT^4}{3}$, $\frac{dP_{\rm rad}}{dr} = -\frac{\rho\kappa}{c}\frac{L_r}{4\pi r^2}$.

using the hydrostatic equation $\frac{dP}{dr} = -\frac{\rho G m}{r^2}$, we get $\frac{dP_{\rm rad}}{dP} = \frac{\kappa L_r}{4\pi c G m}$ (1.20).

- Since L/m typically decreases but κ increases with radius (e.g. for Kramers type κ∝$T^{-3.5}$), he approximated that κ L/m = constant inside the star. Then the right hand side of (1.20) is constant and

$$\frac{P_{\rm rad}}{P} = \frac{\kappa_s L}{4\pi c G M} = \frac{L}{L_{\rm Edd}} = \text{constant} = 1-\beta \quad (1.21).$$

- Therefore, this approximation leads β=constant, and from the Eq. (1.18), the star becomes an n=3 polytrope. This is the Eddington's standard model.
- From (1.21), L = (1-β) L_{Edd} (1.22)

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Eddington's standard model

■ **n=3 Polytrope**

For the polytropic EOS, $P = K\rho^{1+\frac{1}{n}}$, the stellar mass M satisfies the following relation,

$$M = C_1\rho_c^{\frac{3-n}{2n}}, \quad C_1 = 4\pi\left(-\frac{w'}{z}\right)_{z_n} z_n^3\left(\frac{n+1}{4\pi G}\right)^{3/2} K^{3/2} \quad (1.23),$$

where ρ_c is the central density, $w' \equiv dw/dz$, and z_n is the solution of the Lane - Emden equation at $\rho = 0$.

In the Eddinton's model, $n = 3$ and $K = \left(\frac{3k^4}{a\mu^4 m_u^4}\right)^{1/3}\left(\frac{1-\beta}{\beta^4}\right)^{1/3}$.

Therefore, we get an equation, $$M = \frac{18.1M_\odot}{\mu^2}\left(\frac{1-\beta}{\beta^4}\right)^{1/2} \quad (1.24) \quad \text{or}$$

$$1-\beta = 0.00040\left(\frac{M}{M_\odot}\right)^2\left(\frac{\mu}{0.61}\right)^4\beta^4 \quad (1.25). \quad \text{(Eddington's quartic equation)}$$

This equation shows that very massive stars (M >>100$M_\odot$) has β << 1 and Radiation dominated. From (1.22), L = (1-β) L_{Edd} and the luminosities of very massive stars are roughly the Eddington luminosity.

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